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An Evaluation of Various Generalised Autoregressive Conditional Heteroskedasticity Models

What makes a model superior in Estimation and Forecasting UK Stock Market's Volatility

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Abstract

A volatility model must be able to capture the dynamics of volatility accurately. It should perform well in both estimation and forecasting volatility. The literature has yet to agree on the source of difference amongst various volatility models. Therefore, this research seeks the drivers of the superiority of the selected GARCH-family in modelling the volatility of the FTSE 100 index over 2000-2020. It considers if there are models that systematically outperform and if so, which conditions make those models superior over the other ones. An iterative process of identification, estimation and diagnostic tests of Box-Jenkins has been employed in conducting the statistical tests under different scenarios. A wide range of evaluation criteria including the value of log-likelihood, three types of information criteria, Mincer–Zarnowitz series, and different loss functions are employed in terms of both in-sample and out-of-sample performance evaluation. It is revealed that, firstly, the volatility of the UK's stock market is clustering, sensitive to shocks, and persistent with a half-life of 14.5 weeks, still mean-reverting at some point albeit. Secondly, the condition of the market alters the performance of the models, improving after excluding the crisis time from the time series. Additionally, negative innovations have more impact on the volatility compared to the positive shocks so within the GARCH framework, asymmetric models better fit the volatility of the UK market. Moreover, the E-GARCH is the best fitting model among the selected models and its good in-sample performance may translate into a good out-of-sample performance as well. Finally, the implied volatility index of the FTSE 100 (VIX) has incremental information over the GARCH specification. As an output of the research, the inclusion of both asymmetry terms and exogenous variables of VIX into the GARCH-family models are suggested as significant sources of improvement in the quality of a stationary GARCH process. However, employing a different distribution function of t-student is not a meaningful source of difference contrary to what several studies suggest.

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Table of Contents

List of Charts

List of Figures

List of Tables

List of Abbreviations

AC	Autocorrelation Function
AIC	Akaike Information Criteria
APARCH	Asymmetric Power ARCH
AR	Autoregressive
ARCH	Autoregressive Conditional Heteroskedasticity
ARMA	Autoregressive Moving Average
ARIMA	Autoregressive Integrated Moving Average
BOE	Bank of England
BSE	Bombay Stock Exchange
E-GARCH	Exponential GARCH
FTSE 100	Financial Times Stock Exchange 100 Index
GARCH	Generalised Autoregressive Conditional Heteroskedasticity
GJR-GARCH	Glosten, Jaganathan, and Runkle GARCH
HQC	Hannan-Quinn Criterion
LSE	London Stock Exchange
MAE	Mean Absolute Error
MAPE	Mean Absolute Percentage Error
ML	Maximum Likelihood
MZ	Mincer–Zarnowitz
NSE	National Stock Exchange
OLS	Ordinary Least Square
PAC	Partial Autocorrelation Function
RMSE	Root Mean Square Errors
SBC	Schwartz Criterion
SV	Stochastic Volatility
SVI	Internet Search Volume Index
TARCH	Threshold ARCH
VIX	Volatility Index

Chapter 1. Introduction

1.1. Background

Volatility, described as a degree of dispersion of random variables and traditionally measured by the standard deviation of expected returns, is a significant subject in financial and economic studies. It indicates the uncertainty of markets and economies and influences the decision-making process of entities and individuals (Bhowmik and Wang 2020). Stock market volatility affects the real economy through several mechanisms. Any changes in stock market variance influence the investors' behaviour in terms of holding risky assets. The stability of financial markets also impacts the stability of the economy so that volatility is a concern for policymakers and governments (Arnold 2012; Pilbeam 2018). Volatility also has significant implications for financial market participants and finance professionals. A Forward-looking estimation of stock market returns' fluctuations is a key input for asset pricing models e.g., the option pricing model of Black-Scholes-Merton and its extensions, risk management and hedging strategies e.g., Value-at-Risk models, portfolio construction decisions, investment valuation, and trading strategies (Hull 2012; Bhowmik and Wang 2020).

Although a variety of tools and techniques have been introduced so far for modelling and forecasting volatility, the complexities inherent in the process make it significantly challenging. Different models have exhibited various qualities of performance depending on the markets' situation. To specify volatility estimation models accurately, it is necessary to consider the certain properties and patterns of financial time series so-called stylized facts about financial data. Some important stylized facts about volatility are that volatility exhibits heteroscedasticity, clustering and persistence, leptokurtosis, and asymmetric behaviour. Additionally, innovations have an asymmetric impact on volatility (Engle and Patton 2007; Satchell and Knight 2011; Hussain et al. 2019).

Traditional models assume constant volatility. One of the key assumptions in classic regression models is that the variance of error terms is constant over time, known as homoscedasticity. The reality in financial time series is that the residuals are heteroscedastic, and volatility has a time-varying and latent nature (Sharma 2015).

To model time-varying conditional variance and deal with heteroscedasticity, Engle (1982) introduces a stochastic process called autoregressive conditional heteroscedasticity (ARCH) to overcome the implausibility of the assumption for forecasting in traditional models that implied there was a one-period, unconditional, constant variance. In his work, mean and variances of inflation in the UK are estimated and the study shows that the ARCH effect is significant among disturbances and the estimated variance appears to increase during the volatile period of the 1970s. The study uses past disturbances to model the variance using an ARCH model. The ARCH effectively models the heteroscedasticity by defining the conditional variance of the residuals as a linear function of the squared residuals in the previous period (Sharma 2015). Engle (1982) also indicates that the ARCH models provide a better estimate when the Maximum Likelihood (ML) method is used instead of the Ordinary Least Square (OLS) method (Obeng 2016).

The ARCH model required many parameters to be estimated. Furthermore, (Bollerslev 1986) generalizes and extends the ARCH to the GARCH effectively by extension of the Autoregressive (AR) process to an Autoregressive Moving Average (ARMA) process and benefits from its better performance in forecasting the uncertainty of inflation rates. The GARCH defines the conditional variance as a function of its lagged values and squared lagged values of residuals (Sharma 2015; Brooks 2019). Both the ARCH and the GARCH meet the stylized facts however the latter needs a lower number of estimated parameters and therefore is more parsimonious (Damodar N 2004; Sharma 2015).

Another stylized fact that has appropriately been addressed by the standard GARCH models is that financial time series face volatility clustering and leptokurtosis (see Mandelbrot (1997) as one of the earliest works). It implies that large movements in stock volatility are followed by subsequent large changes, and small movements are observed to be followed by small movements in either sign.

Another empirically-proved characteristic is that not only financial markets' returns are negatively skewed and follow fat-tail distributions, but also their volatility does not respond to negative and positive shocks symmetrically (see Black (1976) for one of the earliest works and). This phenomenon is sometimes called the leverage effect and sometimes risk premium effect (Engle and Patton 2007; Brooks 2019). However, this asymmetry has not been considered in the standard ARCH and GARCH models because these models in nature only deal with the magnitudes of the shocks not the positivity or negativity of the shocks.

A response to the failure of standard GARCH models in accounting for asymmetry is the development of different extensions and variants of GARCH. These models have been evolved to address different aspects of the complexity of volatility estimation and prediction (Brooks 2019). Some of the variants include the Exponential-GARCH (EGARCH) model by Nelson (1991), the Glosten, Jaganathan, and Runkle model (GJR-GARCH) modelled by Glosten et al. (1993), the Asymmetric Power ARCH (APARCH) model by Ding et al. (1993), Threshold ARCH (TARCH), and the Threshold GARCH introduced independently by Zakoian (1994), and the Power ARCH model generalised by Ding et al. (1993) (Granger and Poon 2001; Alberg et al. 2008; Sharma 2015; Obeng 2016; Wang et al. 2020).

A valid question here is that what makes these different models more appropriate and accurate in modelling and forecasting volatility. A growing body of literature seeks to compare, contrast, and evaluate the accuracy of these various models. The conclusions imply that selecting the most suitable GARCH-type model to address the market volatility is a challenging task, conditional heteroscedasticity models are among the best available models, and the performance of models is significantly different for various asset classes and different markets (Brooks 2019). However, relatively less work has been done around the possible sources of difference among various models (Chkili et al. 2014; Brooks 2019; Dixit and Agrawal 2019).

1.2. Research Aim

This research aims to evaluate the performance of selected GARCH-type models in estimation and forecasting weekly volatility of the UK stock market during 2000-2020 and ultimately provide recommendations for the models' improvement.

The FTSE-100 is selected as a proxy for the UK market and the study tries to see if there are models that systematically outperform the other ones and if so, which conditions make some models superior over the other ones. The output of the research is recommendations around what makes a volatility estimation model a good model for both estimation and forecasting purposes.

1.3. Research Objectives

Following the aim of the research, the objectives of the research are as follows.

1. To evaluate the ability of standard GARCH, Exponential GARCH, and the GJR-GARCH in modelling and forecasting the weekly volatility of FTSE-100.
2. To find if non-linear GARCH models can systematically outperform the standard GARCH model and if so, what is the best-fitting model among two variants of the non-linear GARCH models.
3. To compare the accuracy of models in the normal course of the economy with the crisis periods and examine if distressed markets' characteristics alter the performance of models.
4. To explore possible sources of improvements in models to make recommendations that benefit both future empirical works and stock market professionals.

This study contributes to the existing literature by more thoroughly examining the sources of difference among GARCH-type models driving the forecast superiority and presents recommendations to improve the models. Additionally, it tries to examine the impact of market conditions on the forecasting performance of GARCH models and since it is solely focused on the UK stock index (FTSE 100), provides an insight into that specific market. Furthermore, it covers a relatively long time among the volatility forecasting studies. This study also uses a wide range of performance evaluation criteria compared to some earlier studies. This study could be beneficial for both researchers and market participants.

The remainder of this research is organised as follows: In Chapter 2, a literature review is presented. Chapter 3 describes the research methodology and data. In Chapter 4, findings and empirical results are presented and discussed, and finally, Chapter 5 presents the conclusion, recommendations for improvement of the models, limitations of the research, and the horizon for future studies.

Chapter 2. Literature Review

Finance and investment literature categorises volatility and risk modelling into different types of models: the Autoregressive Moving Average (ARMA) models, the Stochastic Volatility (SV) models, the Regime-switching models, The Threshold models, and the Autoregressive Conditional Heteroscedasticity (ARCH) models (Brooks 2019). Considering the aim and objectives of this research, this chapter concentrates on the last category namely ARCH and its generalised type called GARCH models. The focus in this literature review is on volatility in financial markets with a special emphasis on forecasting. The first section of this chapter reviews the evolutionary process and development of ARCH models in response to the specific characteristics of financial markets (stylised facts) and then briefly points to the studies testing the stylised facts of financial markets using the ARCH and GARCH family models. The importance of testing the stylised facts of financial data is that firstly, autoregressive conditional heteroscedasticity models have been developed in an attempt to account for different stylized facts (Bhowmik and Wang 2020), and secondly and more importantly, this research seeks the drivers of the superior ability of models in forecasting volatility and a prerequisite for a model to have an explanatory and predictive power is that the model fits the basic characteristics of the specific market being examined.

The second section of the literature review critically compares the ability of GARCH-type models in modelling and forecasting volatility. This section itself is divided into two sub-sections. The first one presents evidence in favour of more developed and sophisticated models as those are expected to be superior based on their complexity and novelty, and the second sub-section critically evaluates if the complexity added to the models improves their performance proportionally. Interestingly, several studies show the opposite of this intuition.

The last section of this chapter reviews the studies seeking a response to the question “what makes a GARCH model a good system in terms of volatility modelling and forecasting?”. Few studies have been done in this area and those limited factors mentioned as drivers of the superiority of models, most of the time are by-products of the studies evaluating the performance of the models. The results are mixed and inconclusive and there is not any consensus in response to this critical question. Considering this fact, this research tries to present some recommendations to improve the ability of GARCH models in estimation and forecasting volatility of the UK stock market.

2.1. Evolutionary Process of Autoregressive Conditional Heteroscedasticity Models

After being introduced by Engle (1982), and being extended and generalised by Bollerslev (1986), the time-varying volatility estimation and forecasting have been conducted for different asset classes and data sets ranging from inflation, interest rates, and exchange rates to stock market indices and cryptocurrencies (Engle 1982; Bollerslev 1986; Chu et al. 2017; Costa 2017). One of the main reasons for the popularity of these models is that many empirical studies confirm their ability to capture the dynamics of conditional variance. In one of the earliest works in this field, the accuracy of GARCH models in comparison with the previous systems has been proved by Akgiray (1989) (SEKMEN and Ravanoğlu 2020). It is also confirmed that the out-of-sample forecasting performance of GARCH models for a one-week horizon is more accurate compared to the previous methods using five exchange rates against the dollar during 1973-1989 (West and Cho 1995; Bhowmik and Wang 2020).

The good performance of GARCH models is attributable to their ability in capturing the common characteristics of volatility. It is empirically confirmed that volatility is clustered, persistent, and conditional, and the GARCH models are proved to provide a good first approximation to the volatility clustering, volatility persistence, nonlinearity and observed temporal dependencies (Engle and Bollerslev 1986; McCurdy and Morgan 1988; Baillie and Bollerslev 1989; Hsieh 1989; Andersen and Bollerslev 1998; Joshi 2010; Li and Wang 2013; Francq and Zakoian 2019; Bhowmik and Wang 2020).

There is also obvious evidence of asymmetry and leverage effect in financial markets meaning that negative shocks increase the volatility more than positive shocks. In a recent study, Aliyev et al. (2020) employ GARCH, EGARCH and GJR-GARCH to estimate the volatility of Nasdaq-100 using daily data over 2000-2019. The result of the study suggests the persistency of volatility shocks and the presence of asymmetry and leverage effect.

In addition to the US market, the leverage effect has been confirmed in several regions. Amongst many other studies, Olowe (2009) confirms that volatility is persistent and there is a leverage effect in the Nigerian stock market. Chang et al. (2011) confirm the asymmetry of the volatility of the Taiwanese financial markets. Abdalla and Suliman (2012) confirm it in the Saudi stock market. Hou (2013) indicates the asymmetric effect of negative news in Chinese stock markets. Okicic (2014) shows the existence of the leverage effect in Central and Eastern European stock markets. Banumathy and Azhagaiah (2015) capture the asymmetry of the Indian stock market during 10 years of 2003-2012 using daily closing prices of the Indian market index (Olowe 2009; Chang et al. 2011; Abdalla and Suliman 2012; Hou 2013; Okicic 2014; Banumathy and Azhagaiah 2015; Bhowmik and Wang 2020).

Although Both the ARCH and GARCH models capture leptokurtosis and volatility clustering, they fail to model the leverage effect because their model specification is symmetric (Alberg et al. 2008; Francq and Zakoian 2019).

Since standard GARCH methods were not able to address the leverage effect, forecasting conditional variance using non-linear and asymmetric GARCH models has been conducted in several papers. According to Fan et al. (2003); Tsay (2005); Sharma (2015); and Bhowmik and Wang (2020), the asymmetric effect of the negative return shocks is captured by various models. To incorporate the asymmetries, the Exponential-GARCH (EGARCH) introduced by Nelson (1991) models the logarithm of the volatility rather than its level. The GJR model (Glosten et al. 1993) incorporates an extra variable to adjust the model upward for the volatility of negative shocks. The Threshold (TGARCH) model introduced by Zakoian (1994) also tries to capture different effects of good news and bad news on volatility. The N-GARCH model (Higgins and Bera 1992), the APARCH model (Ding et al. 1993), and the HGARCH model (Hentschel 1995) are also used to handle asymmetries.

Many other extensions and variants have been evolved to better capture the volatility structure and dynamics of the financial markets. Granger and Poon (2001) in a comprehensive literature review compare the findings of 72 written works in forecasting volatility using different techniques. They address problematic areas of forecasting, the influences of the frequency of data used, the measures selected as proxies for actual volatility, and the effects of crises and adverse events.

In another systematic review recently Bhowmik and Wang (2020) review papers written from 2008 to 2019 using GARCH models to model market volatility and return. They conclude that the GARCH models provide better results combined with other techniques. Based on their study, under symmetric information, volatility can be better explained by GARCH (1,1) and the asymmetric GARCH models could model the volatility more precisely under asymmetric information conditions.

Considering the evolutionary process of the GARCH models in response to the stylised facts about volatility, the current dissertation firstly examines the stylised facts on the selected time-series which is the UK's FTSE 100 index over 2000-2020 to see if the GARCH models are generally suitable to model the volatility of the UK stock market. In the next step, the study compares the performance of a simple GARCH with two more sophisticated asymmetric selected models namely the Exponential-GARCH and the GJR-GARCH to find the best-fitting model among the three. This comparison has been extensively done in the literature. The next sections review some of the marked works in this area.

2.2. Complexity or Parsimonious; Asymmetric GARCH Versus Classic GARCH

A critical discussion has emerged in the literature in terms of questioning the addition of more complexity and dimension into the volatility models. The question is whether the sophisticated models adding more parameters, perform superior in comparison with simpler and more parsimonious models or not.

The literature seeks to compare and evaluate the accuracy and predictive power of these various models. Several studies have been conducted to evaluate the forecasting performance of GARCH-type models in terms of their in-sample and out-of-sample forecasting accuracy. Hansen and Lunde (2005) compare 330 ARCH and GARCH-type models to evaluate their ability in forecasting the one-day-ahead conditional variance. Using exchange rate data and stock market return data, the models are evaluated in terms of out-of-sample performance. The criteria for evaluating models' performance are six different loss functions. In the analysis of exchange rates, they find no evidence that sophisticated and asymmetric models outperform GARCH. However, in terms of stock market data, the GARCH underperforms models that consider the leverage effect. This is a finding in support of the idea that different markets show different volatility dynamics and best-fitting models could be chosen firstly considering market type and condition (Granger and Poon 2001; Hansen and Lunde 2005; Bhowmik and Wang 2020).

The remainder of this section differentiates findings in favour of Asymmetric GARCH from the findings in favour of Classic GARCH models.

2.2.1. Findings in Favour of Asymmetric GARCH models

It is intuitive that since financial markets show leverage effect and asymmetry, asymmetric models better capture and model volatility. The superiority of exponential GARCH has been confirmed in several studies. The E-GARCH outperforms the simple GARCH for volatility forecasts of the US stock stocks (Pagan and Schwert 1990; Hansen and Lunde 2005; Sharma 2015; Lin 2018).

The quadratic GARCH (Q-GARCH) outperforms during the crash of 1987. The Q-GARCH is expected to be able to handle the asymmetric effects of positive and negative shocks. Franses and Van Dijk (1996) evaluate the performance of GARCH and its two variants including Quadratic-GARCH and GJR-GARCH in forecasting the weekly volatility of stock markets in Germany, The Netherlands, Spain, Italy, and Sweden in 9 years. Their study shows that the Q-GARCH outperforms when there are extreme observations, such as the crash of 1987. The criteria used for performance evaluation in this study are the median of squared error as the loss criterion, and it is concluded that the Q-GARCH and GARCH models provide better out-of-sample forecasts than the GJR model (Franses and Van Dijk 1996; Granger and Poon 2001; Sharma 2015).

Using monthly time series in Australia, asymmetric GJR-GARCH is superior. The study compares a wide range of models including a random walk model, a historical mean model, a moving average model. The evaluation criteria are the root mean squared error (RMSE), mean absolute error (MAE) and mean absolute percentage error (MAPE) loss criteria (Brailsford and Faff 1996; Roni et al. 2017).

Another study examines the performance of four GARCH (1,1) models including GARCH, E-GHARCH, GJR, APARCH using three distributions including Normal, Student-t, and Skewed Student-t in two major European stock indices, FTSE 100 and DAX 30. The result is that GJR and APARCH better forecast the volatility using daily data over 15 years in the two markets (Peters 2001; Bhowmik and Wang 2020).

In another work, it is revealed that for forecasting the volatility of the US T-bill yields, the asymmetric GARCH outperform (Bali 2000; Sharma 2015). It is also documented that E-GARCH is the best model for forecasting the exchange rate volatility (Balaban 2004; Sharma 2015).

A volatility forecast for the S&P 500 index futures using the GARCH, E-GARCH and T-GARCH models shows that E-GARCH provides the most accurate forecasts of the future-realized volatility (Bali and Demirtas 2008; Sharma 2015).

Lama et al. (2015) indicates the superiority of E-GARCH over classic GARCH and autoregressive integrated moving average (ARIMA) in modelling and forecasting the behaviour of agricultural commodities prices. The criteria are root mean square error (RMSE) and relative mean absolute prediction error (RMAPE). Their study supports the theory that the E-GARCH model can better capture asymmetric volatility. Agricultural commodities' price is volatile and noisy in nature and this way it is comparable with stock markets. The prices also appear to reveal asymmetric behaviour (different responses to recession and recovery).

The forecasting ability of several GARCH models (GARCH, E-GARCH, GJR and APARCH) have also been examined using different distributions functions (normal distribution, Student's t-distribution and asymmetric Student's t-distribution) for two Tel Aviv stock index returns. The result of the study indicates that the asymmetric GARCH with fat-tail densities are the most promising models in forecasting future volatility. The E-GARCH model using a skewed Student-t distribution is the most successful model (Alberg et al. 2008; Wang et al. 2020).

Using Standard & Poor's 100 stock index and evaluating by the Superior Predictive Ability (SPV) test, it is indicated that the GJR-GARCH model achieves the most accurate daily volatility prediction from 1997 to 2003, closely followed by the EGARCH model (Liu and Hung 2010; Lin 2018).

Using daily stock index return data from Romania (Bucharest Exchange Trading index) from 2001 to 2012, it is concluded that the TGARCH model is the most accurate in forecasting volatility. This study employs three asymmetric models (T-GARCH, E-GARCH, and PGARCH), a wide range of error distribution assumptions for the error terms (the normal distribution function, the Student-t distribution function, and Generalized Error Distribution), and a relatively long period (Gabriel 2012; Drachal 2017).

Hu (2019) conducts a study for different time horizons, between various developed and emerging markets, to find out which one is better for the short time horizon and which one for the longer one. ARCH, GARCH, GJR-GHARCH, C-GARCH, APARCH, GARCHX are the models in order to forecast out-of-sample conditional variances of stock markets. The performance is evaluated using multiple variance proxies including realized volatility, range-based volatilities, MSE, and Q-like loss functions. The result is that GARCHX that incorporates the volatility index (VIX) is superior at multiple horizons for S&P 500. For the remaining indices, mostly CGHARCH and APARCH are better.

Dixit and Agrawal (2019) conduct a study using daily data of Bombay Stock Exchange (BSE) and National Stock Exchange (NSE) from 2011 to 2017. The outcome suggests that the P-GARCH model of (Ding et al. 1993) is the most suitable to predict and forecast the stock market volatility for both markets. P-GARCH model is an extension and combination of other GARCH models to capture the leverage effect.

Åstrand (2020) in a master's thesis examines the performance of symmetric and asymmetric GARCH-family models in forecasting volatility during and post the financial crisis of 2008. The selected models are the basic GARCH and Integrated GARCH as symmetric and E-GHARCH, T-GHARCH, N-GHARCH, and APARCH as asymmetric models. The frequency is daily, and it is to evaluate the return of the OMX Stockholm 30 index against the realized volatility of the market as a proxy. All models underperform the real realized volatility and the difference among the models are small and Exponential GARCH outperforms the post-crisis period and I-GARCH and N-GHARCH are superior during the crisis.

Overall, theoretically, it seems intuitive, and it also empirically has been supported by many studies that the more sophisticated models, by the inclusion of asymmetry variables in their models, better model the volatility. But is it that obvious?

2.2.2. Findings in Favour of Classic GARCH models

Although sophisticated models try to better capture the empirically approved stylized facts of the volatility, it is not always the case that the forecasting performance of the more sophisticated GARCH-family is impressive. In some cases, the findings are inconclusive and there is found no difference between the GARCH and asymmetric types of it (Ederington and Guan 2005; Sharma 2015). In some other cases, the simple GARCH models surprisingly perform best. Even if a heavily parameterised model could outperform the simpler models in terms of in-sample fitness, on the out-of-sample forecasting ability, the simpler models sometimes outperform the more advanced ones (Balaban 2004; Sharma 2015).

In a study, the E-GARCH is outperformed by the other time series models for the long term forecasts (Cao and Tsay 1992; Sharma 2015). The Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) underperforms during the crash of 1987 (Franses and Van Dijk 1996; Granger and Poon 2001; Sharma 2015).

Hou (2013) in an attempt to see whether the advanced GARCH models outperform the standard GARCH model in forecasting volatility, concludes that the GJR models overestimate the volatility under a high-volatile condition.

Sharma (2015) finds that the standard GARCH model beat the more complex GARCH models and provides the best one-day-ahead forecasts of the variance. Comparing the daily variance forecasts of seven GARCH-type models for 21 stock markets of the world for the period 2000 to 2013 using multiple statistical tests, this study claims that the results are dependent on the selection of performance evaluation criteria and different market conditions.

Obeng (2016) in a study for the forex market finds that the simple GARCH outperforms all the other models and then claims that special consideration should be taken on the simple models. This study investigates the forecasting power of ARCH, GARCH and E-GARCH models in forecasting exchange rate volatility in terms of both in-sample and out-of-sample performance evaluation using loss functions.

To detect which is the best model among the GARCH-type model, (Costa 2017) using the Superior Predictive Ability (SPA) test from Hansen and Lunde (2005) and series of Mincer-Zarnowitz regressions, indicates that the asymmetric GARCH models cannot outperform the simple GARCH in forecasting next day conditional volatility. The focus of this research is on the NASDAQ-100 in the US market.

Bhowmik and Wang (2019) using daily, weekly, and monthly closing prices of six Asian emerging stock market indexes from 2007 to 2016 compare ARMA and GARCH based on symmetric error statistics. The study suggests that the simple GARCH models appear to have the best fit for the data.

Considering the empirical finding which is in contradiction with the assumption that the sophisticated is better, the question is what are the sources of difference among different models? The next section of the literature review tries to find a response to this question and this concept effectively is the pivotal point of the current dissertation.

2.3. What Makes a GARCH-type model powerful?

A volatility model as the name suggests must be able to model and forecast volatility accurately. Theoretically, the more appropriate the common properties of financial time series could be captured by the model, the more accurate the model could be. That is the reason why the literature is dominated by examining the fitness of the models to the stylised facts of financial time-series.

Engle and Patton (2007) in their paper outline the stylised facts about volatility that should be incorporated into a model. They use the Dow Jones Industrial index data to examine these stylised facts, and the ability of GARCH variants to capture these features (Engle and Patton 2007; Francq and Zakoian 2019).

Based on both theory and the literature, for a volatility model to be an accurate model, firstly it should fit properly to in-sample dataset (capture the properties of the market with minimum estimation errors during the sampling period), and secondly, the model should work properly for any dataset other than the sampling period as well. In other words, the model should have forecasting power. One of the most dominant recommendations for the improvement of volatility models is that good forecasting performance needs a specification that can include the asymmetry of shocks and the leverage effect (Hansen and Lunde 2005; Bhowmik and Wang 2020).

However, some authors have been critical of the predictive power of GARCH-type models. They claim that these models are solely powerful in in-sample parameter estimation but cannot perform well in terms of out-of-sample data and on the ex-post basis. In other words, some models with good in-sample fitness, forecast future volatility poorly, especially on longer horizons (Cumby et al. 1993; Jorion 1995; Huq et al. 2013; Wennström 2014; Brooks 2019).

In contrast, Andersen and Bollerslev (1998) claim that if GARCH-type models are well-specified, they can in turn provide accurate forecasts. They believe that the problematic point is the volatility measurement criteria the models are judged with rather than the models themselves. Using GARCH (1,1) specification of Bollerslev (1986) on Deutschemark-U.S. Dollar and Japanese Yen- U.S. Dollar spot exchange rates, they find that daily ARCH and GARCH models perform well, explaining around 50% of the ex-post variability. They introduce an alternative method using high-frequently intraday data as more meaningful and accurate ex-post volatility measurement criteria. Theoretically, increasing the frequency of observations to an infinitesimal interval causes the most accurate measure for the volatility factor. It is claimed that the models do have predictive power and the problematic part of the process is the measures evaluating their power (Andersen and Bollerslev 1998; Francq and Zakoian 2019). Some other studies had before shown that the out-of-sample forecast performance of the GARCH models depends on the choice of loss evaluation criteria. It is in support of the idea that it is the evaluation measures that are the source of difference, not the models themselves (Sharma 2015).

Nonetheless, the literature includes studies seeking the areas of improvement for volatility forecasting models. Generally, a heavily parameterized model should be better able to capture the dynamics of the volatility compared to a relatively simple model.

Studies document that using asymmetric GARCHs e.g., EGARCH improves the forecasting process of the exchange rate volatility (Balaban 2004; Sharma 2015). Another empirical work investigating the stock market volatility in 10 stock exchanges in Central and Eastern European countries during 1991-2008 confirms that models which allow for the inclusion of asymmetric variables, consistently outperform all other models considered (Alberg et al. 2008; Harrison and Moore 2012; Dixit and Agrawal 2019).

According to different types of studies, a problem while using GARCH models is that they do not appropriately consider the skewness, kurtosis, and fat tails property of financial time series. To overcome this shortcoming, many studies have run models using asymmetric distribution functions rather than normally distributed density curves. These studies assert that the type of distribution function used in the models is a source of difference among various models in volatility forecasting quality. They differentiate the GARCH family models by the distribution function used in the model specification and compare their forecasting performance. The comparison of the normal distribution with non-normal ones shows that the asymmetric GARCH models under Student-t distribution are the best for characterizing the behaviour of the returns including serial correlation, asymmetric volatility clustering and leptokurtic innovations (Alberg et al. 2008; Wang et al. 2020).

The importance of choosing an appropriate density function has also been emphasised in another empirical study conducted by Wennström (2014). To compare the estimation and forecasting performance of volatility models, (Wennström 2014) examines the performance of six models: The simple moving average, the exponentially weighted moving average, the ARCH model, the GARCH model, the EGARCH model and the GJR-GARCH model in three different Nordic equity indices. This study implies that for an in-sample fitness, modelling the conditional mean using a heavier tailed error distribution rather than a normal distribution significantly improves the fit.

However, using S&P 100 stock index and evaluating by the Superior Predictive Ability (SPV) test, it is strongly suggested that in order to improve the predictive power of GARCH family models, incorporating asymmetry components to the model is more important than error distribution specification (Liu and Hung 2010; Lin 2018).

Selmi et al. (2015) suggest the application of Artificial Neural Networks (ANNs) as dealing with the nonlinearity of volatility data is problematic using conventional methods.

A different type of study investigates if the inclusion of exogenous variables has an improving effect on volatility. This category of studies asserts that the volatility of stock markets in addition to being a function of the squared residuals and the lagged volatility of the time-series itself, could be a function of some exogenous financial and economic variables (Engle and Patton 2007; John et al. 2019).

Chronopoulos et al. (2018) indicate that incorporating a variable from Google called Internet Search Volume Index (SVI) in various GARCH family models significantly improves volatility forecasts. The study uses US stock return (a daily frequency S&P500 index covering the period from 2004 to 2016 and daily internet search volume index (SVI) from Google).

Attempting to improve the performance of volatility modelling, a recent stream in the academic literature uses the information content of the “Implied Volatility Index” (VIX) index by incorporating this index as an explanatory variable into the GARCH models’ specification. The logic behind this idea is that VIX is a forward-looking proxy for volatility and carries extra information that helps better capture the dynamics of the volatility of stock markets both in estimation and forecasting. Kanniainen et al. (2014) suggest that the volatilities extracted from VIX on the S&P 500, improves the models’ performance.

Kambouroudis and McMillan (2016) using data from the US, the UK and France stock markets suggest that the inclusion of two independent variables (the VIX and volume) can improve volatility forecasts over a standard GARCH-based model.

Pati et al. (2018) considering the dataset from three Asia-Pacific stock markets including India, Australia, and Hong Kong from 2008 to 2016 confirm that for forecasting stock market volatility, incorporating the VIX can improve the GARCH family model forecasts. In a study, they incorporate implied volatility index as an explanatory variable to the conditional variance equation of the GARCH family model and notice that it reduces volatility persistence and improves the model. As a proxy for volatility in terms of in-sample fit, this study uses the return-based realized variance and the range-based realized variance. To evaluate the forecasting performance, the one-day-ahead rolling forecasts, and the Mincer–Zarnowitz regression and encompassing regression are employed.

Hu (2019) also confirms that the inclusion of a precise volatility proxy of an index to the GARCH model significantly improves the forecast performance.

To summarise the literature review, it should be mentioned that forecasting the forward-looking volatility is not straightforward and evaluating the forecasting performance is even more challenging. Additionally, a good in-sample fit does not necessarily imply a powerful out-of-sample forecasting performance. The literature encompasses all the effort and contributions that have been done so far to recognise the best-fitting and the most powerful models. There is no consensus about the source of difference in the predictive power of the models and the drivers of improvements in the models' performance could vary depending on the market condition, the properties of the underlying asset being traded, and the period being investigated. The normal course of financial markets exhibits different behaviour than adverse events and financial crises, and the superiority of models can alter from one financial and business cycle to another. Volatility is chaotic in nature (Brooks 2019) and modelling and forecasting a chaotic variable is challenging, if not impossible. A rapidly growing body of knowledge in the literature in recent years has emerged employing Artificial Intelligence, Big Data, Unstructured Data Analytics e.g., analysis of the content posted on social media, and Natural Language Processing (NLP) to better capture the dynamics of volatility. This research does not enter this very new area of AI and Big Data, but it seems incorporating these techniques into modelling and forecasting volatility could potentially be a significant source of improvements in the models (Selmi et al. 2015). Considering the theoretical background and the previous empirical works, the current dissertation has selected two widely accepted asymmetric GARCH models and wants to compare their performance together and with the simple classic GARCH and evaluate them with various measures in a relatively long period, to detect the drivers of the most accurate performance. In seeking the sources of difference in the performance of the various models, the current research compares the performance of the selected models in the normal market and the times of crisis. The research also examines the idea of incorporating an exogenous variable (VIX) into the GARCH-family models to see if this could be a potential source of difference. The idea of conducting the models under different distribution functions has also been tested by comparing the performance of the models under a normal distribution and the t-student density function. The outcome of the research is recommendations to improve the GARCH models in forecasting the volatility of the UK stock market. The next chapter of this work describes the methodology under which the research has been carried.

Chapter 3. Research Methodology

This chapter presents the methodology used to address the aim and objectives of the research in evaluating the performance of different GARCH-family in both modelling and forecasting volatility of the UK stock market. The chapter is divided into three sections. The first section concentrates on the research approach, the selected models, the way the models are estimated and forecasted, and the theoretical considerations about the selected methods and the methodology. The second section explains the evaluation criteria for the performance of the selected models in detail. The final section of this chapter also describes the datasets used to conduct the research, their main sources, the sampling period and sub-periods of the research, and the reason behind the selected datasets and time periods.

3.1. Research Approach

This research aims to evaluate the performance of selected GARCH-type models in estimation and forecasting weekly volatility of the UK stock market during 2000-2020. The FTSE-100 is selected as a proxy for the UK market and the study tries to see if there are models that systematically outperform the other ones and if so, which conditions make some models superior over the other ones.

The strategy employed to carry the research is a quantitative analysis using the publicly available weekly datasets of the UK's FTSE 100 index and the volatility index of the FTSE 100. Based on the literature and theoretical background, the first two steps in the volatility model's specification are data inspection and testing for stylized facts of the volatility of financial markets. The goal of these steps is to see if the models fit the selected data over the selected period. The next step is to compare the different models amongst the fitting models in terms of in-sample fit and out-of-sample forecast performance (Satchell and Knight 2011).

The defined process is to take the weekly observations of the FTSE 100 index (I_t) and to calculate its weekly logarithmic return using Equation (1). The log-returns are used in the literature to increase the likelihood that the process is stationary. However, the standard tests for stationarity are applied to make sure the process is stationary.

$$r_t = logI_t - logI_{t-1} \quad (1)$$

An iterative process of identifying, estimating, and checking proposed by the Box-Jenkins approach is applied to the time series analysis (see Anderson (1977) and Makridakis and Hibon (1997) for some of the early explanations). The data are examined for common characteristics of financial series (volatility clustering, leptokurtosis, heteroscedasticity, and autocorrelation in the residuals) before running the models. The data are also tested for the ARCH effect using the Lagrange Multiplier test of Breusch and Pagan (1980) to see if the GARCH-type models are applicable. The examination period is over 2000-2020. However different sub-periods have been examined separately to detect the effects of different market situations.

Residual diagnostic checks have been conducted after running each model to make sure that little Arch effect, serial correlation, heteroscedasticity, and leptokurtosis have been left over. The logic behind this is to see if the models can capture all these effects.

The selected models are the standard GARCH model, the Exponential GARCH model, and the JGR-GARCH model.

3.1.1. Standard GARCH Model

According to the literature (amongst others Franses and Van Dijk (1996) and Bhowmik and Wang (2019)), to model the volatility of returns, the GARCH (1,1) is empirically dominant since it is parsimony compared to the higher orders of GARCH (p,q). Hence the selected order to carry this research is GARCH (1,1).

Based on Brooks (2019), Equation (2) specifies the conditional variance equation which is aimed to be estimated using the GARCH process.

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (2)$$

Where σ_t^2 is the conditional variance, u_{t-1}^2 are the squared lagged residuals (ARCH term), σ_{t-1}^2 is the lagged conditional variance (GARCH term), and $\alpha_i > 0, \beta_i > 0$

To generate the error terms used in Equation (2), the conditional mean equation is defined as a simple equation with solely a constant term.

3.1.2. Asymmetric GARCH Models

Standard GARCH models treat the positive and negative movements of markets equally and in a symmetric way. By incorporating extra terms into the standard model, the asymmetry could be considered. This way the responses to negative shocks will be different from positive shocks, compatible with reality. Two asymmetric models are used in the research. The GJR-GARCH and the Exponential-GARCH.

The GJR-GARCH Model

Based on Glosten et al. (1993) and Brooks (2019), the GJR-GARCH model is specified by Equation (3).

$$\boldsymbol{\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1}} \qquad (3)$$

Where σ_t^2 is the conditional variance, u_{t-1}^2 is the ARCH term, σ_{t-1}^2 is the GARCH term, and $\gamma u_{t-1}^2 I_{t-1}$ is the asymmetry term.

by incorporating an indicator function, $\boldsymbol{I_{t-1}}$, into the model, the asymmetry has been considered. This way the responses to negative shocks will be different from positive shocks which is more realistic and compatible with the empirically proved stylised facts about the volatility of the stock market. $\boldsymbol{I_{t-1}}$ takes zero when the shocks are positive, turning back the GJR-GARCH model to a standard GARCH. It also takes one for negative shocks, adjusting the volatility towards higher amounts due to the leverage effect.

The Exponential-GARCH Model

Based on Nelson (1991), Exponential-GARCH could be specified as Equation (4).

$$\boldsymbol{ln(\sigma_t^2) = a_0 + \beta_1 ln\sigma_{t-1}^2 + \gamma \frac{u_{t-1}}{\sigma_{t-1}} + \theta \frac{|u_{t-1}|}{\sigma_{t-1}}} \qquad (4)$$

Where u_t^2 is the conditional variance, $\theta_i \frac{|u_{t-1}|}{\sigma_{t-1}}$ is the ARCH term, $\beta_1 ln\sigma_{t-1}^2$ is the GARCH term, and $\gamma_i \frac{u_{t-1}}{\sigma_{t-1}}$ is the asymmetry term.

When $\gamma_i < 0$ and at the same time $u_{t-i} < 0$ (there is a negative shock), the third term becomes larger, adjusting the volatility towards higher amounts due to the asymmetry effect. Again, this is more realistic and better captures the dynamics of the volatility of stock markets.

3.2. Evaluation of the Models

The research objectives are addressed through evaluations of performance of these models by using several measures to determine the coefficient's significance and the model's predictive ability. The differentiating point of this research is its wide range of performance evaluation criteria trying to find the drivers of the superiority of models and to provide recommendations to improve the volatility models. The evaluation has been done in two main areas. The first area is the performance of models in estimation (Goodness-of-Fit), and the second one is their ability in forecasting. To compare the fit and forecast performance and accuracy of models, the following criteria are chosen.

3.2.1. In-sample evaluation criteria

The value of the log-likelihood, and three types of Information Criteria including Akaike Information Criteria (AIC) introduced by Akaike (1973), Schwartz Criterion (SBC) introduced by Schwarz (1978), and the Hannan-Quinn Criterion (HQC) introduced by Hannan and Quinn (1979) are selected as in-sample evaluation criteria (Fit performance).

The higher the value of the log-likelihood, the better-performing the model. In terms of the Information criteria, all the criteria including the AIC are required to be lower for a better model.

3.2.2. Out-of-sample evaluation criteria

Conducting a series of Mincer-Zarnowitz regressions, the Root Mean Square Errors (RMSE), the Mean Absolute Error (MAE), the Mean Absolute Percentage Error (MAPE), and the Symmetric MAPE are the criteria selected for the out-of-sample performance evaluation (Forecast performance).

According to Kambouroudis and McMillan (2016), The first criteria, the Mincer-Zarnowitz regressions framework, effectively examines the statistical significance of the difference between the realised volatility and the forecast value. To do so, the literature firstly recommends that a realized volatility should be calculated as a benchmark. A common benchmark is the measure specified by Equation (5) based on Pagan and Schwert (1990) and Franses and Van Dijk (1996).

$$\boldsymbol{\omega_t = (r_t - \bar{r})^2} \tag{5}$$

Where r_t is the return of each week and $\bar{r}$ is the average return.

Furthermore, to examine the forecast accuracy of the model in predicting the realized volatility, it is needed to examine whether the one-week-ahead forecasted amounts using the selected models are significantly different from the realised ones using Equation (5) or not. According to Engle and Patton (2007) and Kambouroudis and McMillan (2016), this is examined by regressing the values of $\boldsymbol{\omega_t}$ from Equation (5) on a constant and the one-week-ahead forecasted variances as Equation (6).

$$\boldsymbol{\omega_t = \alpha + \beta h_t + u_t} \tag{6}$$

Where ω_t is the realised volatility of each week, α and β are the regression coefficients, h_t is the one-week-ahead forecasted variance and u_t is the error term of the regression.

Then two statistical hypothesis tests about the slope and intercept of the regression are conducted to see if the forecasted variable and the realised variable are moving together. The rest of the criteria including RMSE, MAE, MAPE, and Symmetric MAPE are statistical metrics and need to be lower for better models.

3.3. Data

Two datasets are used to carry the research. The main dataset consists of weekly closing values of the FTSE 100 index over 20 years of 2000-2020. The FTSE 100 index is a value-weighted index of the shares of the 100 companies with the highest market capitalization that are traded on the London Stock Exchange (London Stock Exchange 2021). The intention of choosing the 20 years is to include the data from the global credit crisis of 2007, the European sovereign debt crisis of 2011, and the health crisis of 2019 along with the normal times of the economy. The values are the ones recorded on Wednesdays. The source of data is London Stock Exchange (LSE) website's database (LSE 2021). The examination period is split into two sub-periods, the estimation sub-period, and the forecasting sub-period. The estimation sub-period is from January 2000 to mid-September 2007. This period is used to find the best in-sample fitting models among the selected models. The remaining period from mid-September to the end of 2020 is used for the examination of the out-of-sample forecasting accuracy. The mid-September has been marked by the Bank of England (BOE) as the beginning of the financial crisis of 2007 in the UK and that is why the estimation sub-period end at that point. This is chosen to examine if the selected models that have an accurate in-sample performance, can also forecast accurately, especially when it comes to the prediction of the volatility of returns during crises and adverse events. A relatively long out-of-sample period has been included to avoid a falsely optimistic picture of a model's predictive power stem from model over-fitting leading to spurious conclusions. Chart 1 indicates the weekly values of the index over the examination period.

Chart 1: The FTSE 100 Weekly Values Throughout the Study

Another dataset consists of the weekly value of the Implied Volatility Index of the FTSE 100 (VIX). The VIX is a forward-looking measure of the volatility that investors expect to see in the future. It is a benchmark to quantify market expectations of volatility and sometimes is called the “Fear Index” (Kambouroudis and McMillan 2016). This variable is used as an exogenous variable incorporated into the GARCH to examine if can improve the ability of models. The period this variable is used is from the start of 2013 to the end of 2018. This is a period the UK economy relatively has recovered from the financial crisis of 2007, and the European sovereign crisis of 2011, and before it is hit by the health crisis of 2019 meaning this is chosen as a proxy for a relatively normal course of economy and markets. The source of this database is the historical database from the Investing website (Investing Website 2021).

To summarize the methodology, it should be mentioned that the weekly returns of the FTSE 100 are computed as continuously compounded returns on the index over the period t-1 to t using Equation (1) of the research. The iterative process of identification, estimation and diagnostic checks of the **Box-Jenkins approach** is employed. Before starting the estimation and in the **identification phase**, the datasets have been tested for stylised facts about volatility and the ARCH effects. In the **estimation phase**, the selected GARCH-family models have been conducted to compare their goodness of fit over the examination period. The period then has been split into two sub-periods, one for the estimation purpose and the second for the forecasting purpose. The EViews software is used for estimation and forecasting. Based on Damodar N (2004) and Brooks (2019), the estimation method is the maximum likelihood instead of the ordinary least squares (OLS) method. To evaluate the effectiveness of the models both in-sample and out-of-sample, different criteria including the value of the log-likelihood, AIC, SBC and HQC have been used. To evaluate out-of-sample performance, along with the other criteria (RMSE, MAE, MAPE, and the Symmetric MAPE), a one-week ahead static forecast is applied meaning that after each estimation sub-period, a one-week ahead volatility has been forecasted. Then the real data has been added to the dataset, and again a one-week ahead forecast has been conducted. The process has been repeated until the end of the out-of-sample forecasting sub-period. Then a series of Mincer-Zarnowitz regressions is conducted to statistically examine the difference between the realised and predicted volatility. To meet the aim and objectives of the research and ultimately present recommendations for the improvement of the volatility models, the tests have been repeated by excluding the crises times to see the performance of the models in normal times. The models have also been tested by the inclusion of an exogenous variable called VIX to the models to examine if it can improve the performance of the models. Finally, different tests have been run using the t-student distribution function to test the performance of the model under a different distribution function. Finally, in the **diagnostic checks phase** of the Box-Jenkins approach, some common tests have been conducted to make sure the models have captured the dynamics of the volatility adequately. This process has been repeated for all hypothesis tests. Chapter 4 runs the above-mentioned tests and presents the findings, and their analysis and discussion.

Chapter 4. Empirical Results and Analysis

This chapter describes the findings of the research and their analysis and discussion. It is divided into three sections. The first section overviews the descriptive statistics of the FTSE 100 index and tests for the empirically approved stylised facts about financial time series to identify the overall suitability of the GARCH-family models. This section tries to examine if the GARCH-family models are appropriate for the modelling and forecasting of the volatility of the FTSE 100 index (identification phase). The second section conducts the models under different scenarios (estimation phase), shares the finding of the tests, and tries to critically analyse them considering the literature and the theoretical background. The third section of this chapter conducts some statistical diagnostic checks on the selected models to examine their adequacy in modelling and forecasting volatility (diagnostic checks phase).

4.1. Identification

4.1.1. Descriptive Statistics and Tests for the Stylized Facts

Using Equation (1), the weekly logarithmic returns of the FTSE 100 over 2000-2020 have been calculated to convert the time-series to a stationary process. The first step before using GARCH models is to test for the Stylised Facts and see if the dataset is appropriate to estimate the models using the GARCH family. Chart 2 presents a visual presentation of the main time series. The chart seems like a white noise process with some volatility clustering and volatility persistence over the examination period. Before any formal test, it seems that the volatility has more persistence and longer memory around the years 2002, 2008, 2011, and 2019. These times could be approximately specified by some major financial crises namely bursting the dot-com bubble of the USA, the global financial crisis of 2008, the European sovereign crisis of 2011, and the global health crisis of 2019 (Pilbeam 2018).

Chart 2: Weekly Log-Returns of FTSE 100 over 2000-2020

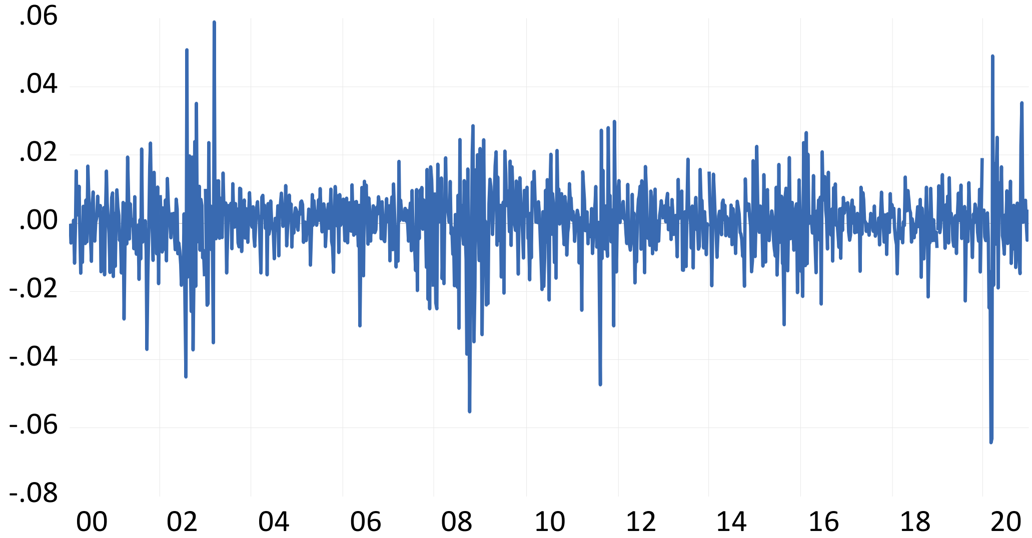


Figure 1 indicates the histogram and statistics of the time series over the examination period. As it shows, the index has a small positive average weekly return of 0.000122% over the period. The weekly standard deviation is also equal to 1.06%, which is much greater than the mean. Median is also a small positive amount. The minimum amount of the return over the period in absolute term is bigger than the maximum return showing the asymmetric range of returns. This is also supported by the evidence for negative skewness. The skewness coefficient from Figure 1 is less than zero indicating that the returns distribution is negatively skewed. This is a common feature of equity returns (Engle and Patton 2007; Brooks 2019). There is also a substantial excess kurtosis (A normal distribution has a kurtosis of 3). The kurtosis coefficient, which is a measure of the thickness of the tails of the distribution, is very high (8.53). This observation is also not strange regarding financial time series.

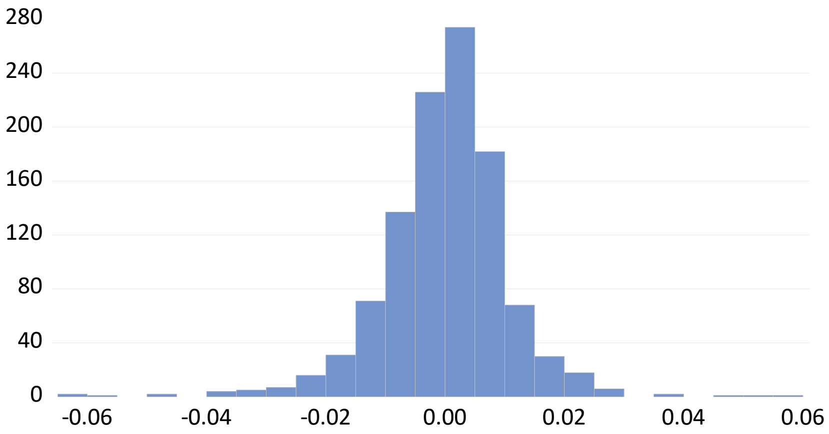
The test statistic of the Jarque-Bera (JB) normality test from Figure 1 is a huge number equal to 1436.792. The null and alternative hypotheses for the normality test are as follows.

H_0: $There\ is\ a\ normal\ distribution$

H_1: $There\ is\ not\ a\ normal\ distribution$

The JB statistic has a Chi-squared distribution with the degrees of freedom of 2. The critical values from the Chi-squared tables at 1%, 5%, and 10% levels for 2 degrees of freedom are respectively 9.2, 5.99, and 4.605. Therefore, the JB is so greater than the critical value at all levels of significance, rejecting the null that residuals are normally distributed. Therefore, the distribution is not normal. This is what was expected based on the literature about empirically approved stylised facts about financial data (Engle and Patton 2007; Brooks 2019). Overall, the data from Figure 1 confirm the common properties of the financial datasets.

Figure 1: Descriptive Statistics and Histogram



Series: FTSE_100_LOG_RETURNS
Sample 1/12/2000 12/30/2020
Observations 1085

Mean	1.22e-06
Median	0.000724
Maximum	0.059010
Minimum	-0.064383
Std. Dev.	0.010633
Skewness	-0.558211
Kurtosis	8.525865
Jarque-Bera	1436.792
Probability	0.000000

4.1.2. Stationarity Test

Using the Augmented Dickey-Fuller test for stationarity (see Figure 2), the ADF test statistic is -35.12 and since it is less than all the critical values at all levels of significance (-3.44 at 1% level, -2.86 at 5% level, and -2.57 at 10% level), it is significant. Therefore, the null hypothesis that implies that the series has a unit root can be rejected and it can be claimed that the series is stationary over the examination period. This is another factor confirming the appropriateness of using the GARCH.

4.1.3. Arch Effects

Using the Lagrange Multiplier test (see Figure 3), both T×R-squared the F-statistic are significant because their test statistics are huge and their p-values are zero. The p-values of zero indicate that the probability that “the null hypothesis is true” is almost zero. Therefore, the null hypothesis that implies that there is no Arch effect can be rejected using both F-statistic and the T×R-squared and the series exhibit evidence of heteroscedasticity (Arch effect) suggesting the use of the GARCH models for volatility modelling of the FTSE 100.

Figure 2: The ADF Test Results

Null Hypothesis: FTSE_100_LOG_RETURNS has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-35.12399	0.0000
Test critical values:	1% level	-3.436165	
	5% level	-2.863996	
	10% level	-2.568129	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(FTSE_100_LOG_RETURNS)
Method: Least Squares
Date: 08/12/21 Time: 14:19
Sample (adjusted): 1/19/2000 12/30/2020
Included observations: 1084 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
FTSE_100_LOG_RETURNS(-1)	-1.065572	0.030337	-35.12399	0.0000
C	1.24E-06	0.000323	0.003860	0.9969

R-squared	0.532754	Mean dependent var	3.88E-06
Adjusted R-squared	0.532322	S.D. dependent var	0.015529
S.E. of regression	0.010620	Akaike info criterion	-6.250375
Sum squared resid	0.122025	Schwarz criterion	-6.241171
Log likelihood	3389.703	Hannan-Quinn criter.	-6.246891
F-statistic	1233.694	Durbin-Watson stat	2.004294
Prob(F-statistic)	0.000000		

Figure 3: The Heteroskedasticity Test

Heteroskedasticity Test: ARCH

F-statistic	271.6691	Prob. F(1,1082)	0.0000
Obs*R-squared	217.5490	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 08/12/21 Time: 15:17
Sample (adjusted): 1/19/2000 12/30/2020
Included observations: 1084 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	6.24E-05	8.97E-06	6.959797	0.0000
RESID^2(-1)	0.447978	0.027179	16.48239	0.0000

R-squared	0.200691	Mean dependent var	0.000113
Adjusted R-squared	0.199952	S.D. dependent var	0.000310
S.E. of regression	0.000277	Akaike info criterion	-13.54041
Sum squared resid	8.33E-05	Schwarz criterion	-13.53120
Log likelihood	7340.900	Hannan-Quinn criter.	-13.53692
F-statistic	271.6691	Durbin-Watson stat	1.992428
Prob(F-statistic)	0.000000		

4.1.4. Autocorrelation Effects

The correlogram of the series' return also shows evidence of autocorrelations in some lags confirming the need for the autoregressive conditional heteroscedasticity models to capture the autocorrelations (see Figure 4). According to Brooks (2019), a good approach to realise autocorrelation is to construct a confidence interval. At the level of significance of 5%, the non-rejection area will be:

$$\pm 1.96 \times \frac{1}{\sqrt{T}} = \pm 1.96 \times \frac{1}{\sqrt{1085}} = \pm 0.059$$

Where T is the sample size equal to 1085 observations.

The amounts of Autocorrelation Function (AC), and the Partial Autocorrelation Function (PAC) should be compared with the non-rejection area (± 0.059). Figure 4 shows that the amounts of AC and PAC for lags 1 and 7 are outside the confidence interval and this is the evidence of Autocorrelation for the 7th lag (for the first lag this is normal to be outside the area). This confirms the need for the autoregressive conditional heteroscedasticity models to capture the autocorrelations.

Figure 4: Q-stat, AC, and the PAC of Standardised Return

Date: 08/12/21 Time: 14:34
Sample: 1/12/2000 12/30/2020
Included observations: 1085

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.066	-0.066	4.6769	0.031
		2	-0.031	-0.036	5.7329	0.057
		3	0.006	0.001	5.7707	0.123
		4	-0.049	-0.050	8.3685	0.079
		5	0.041	0.035	10.195	0.070
		6	-0.022	-0.021	10.731	0.097
		7	-0.084	-0.085	18.517	0.010
		8	0.044	0.030	20.666	0.008
		9	-0.004	-0.001	20.684	0.014
		10	-0.049	-0.051	23.302	0.010
		11	-0.027	-0.040	24.087	0.012
		12	-0.008	-0.007	24.166	0.019

To conclude this section, the identification phase of the Box-Jenkins approach, the descriptive statistics and the examination of the general properties of the time series shows that the GARCH family models are generally appropriate for the research objectives since the data meet the general prerequisites of the GARCH models. The next section runs the selected models in different sub-periods and under different scenarios to address the research aim and objectives (estimation phase of the Box-Jenkins approach). For the ease of following the text, the next section is organised in the same order as the research objectives are defined.

4.2. Estimation and Empirical Results

The first objective of the research is to evaluate the ability of standard GARCH, Exponential GARCH, and the GJR-GARCH in **modelling** and **forecasting** the weekly volatility of FTSE-100. To address this objective, the examination period has been split into different sub-periods. All the basic tests presented in Section 4.1 of this research are also repeated for every single sub-period being examined. The results indicating the suitability of GARCH-family methods for all sub-periods are reported in Appendix B. The two main sub-periods are the estimation sub-period containing data from the January of 2000 to mid-September of 2007, and the forecasting sub-period from mid-September of 2007 to the end of 2020. The models are run for both estimation and forecasting sub-periods and are evaluated using the specified criteria. The second objective of the research is to find if non-linear GARCH models can systematically outperform the standard GARCH model and if so, what is the best-fitting model among two variants of the non-linear GARCH models. Sections 4.2.1 and 4.2.2 presents the findings of the research regarding the first and second objectives.

The third objective is to compare the accuracy of models in the normal course of the economy with the crisis periods and examine if distressed markets' characteristics alter the performance of models. To address this objective, another sub-period is selected. This is from the beginning of 2013 to the end of 2018. This is a period the UK economy relatively has recovered from the financial crisis of 2007 and the European sovereign crisis of 2011, and it is before hitting by the health crisis of 2019 (Bank of England 2021). This period is chosen as a proxy for a relatively normal course of economy and markets. This objective is addressed in Section 4.2.3 of this research.

The fourth and final objective of the research is to explore possible sources of improvements in models to ultimately provide recommendations benefitting both future empirical works and stock market professionals. To address this objective, firstly the models are evaluated after incorporating an exogenous variable called the implied volatility index of the FTSE 100 (the VIX). Secondly, the models are compared using t-student and normal distribution functions. And finally, the impact of jointly inclusion of the VIX and using the t-student function are examined in the improvement of the selected models. The subperiod of the inclusion of the VIX variable and testing under different distribution functions is also the normal period of the market. This way the performance of the models can be assessed after effective controls for the influences of the financial crises. This objective is addressed in Section 4.2.4 of this research.

4.2.1. Evaluation of the In-sample Performance of the Models

The estimation sub-period is covering the January of 2000 to mid-September of 2007 and it tests the in-sample performance of the selected models. After running the tests explained in Section 4.1 confirming the series is stationary, there is Arch effect, and the stylised facts are confirmed in this sub-period (see Appendix B1), the three different GARCH models (standard GARCH, the GJR-GARCH, and the Exponential-GARCH) have been run. The lag order for all three models is one. The statistical results of the estimation are summarised in Table 1. The detailed reports of the EViews software are presented in Appendix C.

Table 1: The Results of the Model Estimations over the Estimation Sub-period

Model	Specification	Variables	Coefficients		P-value
GARCH	$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2$	Constant	α_0	0.00000618	0.0018
		u_{t-1}^2	α_1	0.227	0.0000
		σ_{t-1}^2	β_1	0.723	0.0000
GJR-GARCH	$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1}$	Constant	α_0	0.0000134	0.0000
		u_{t-1}^2	α_1	-0.111	0.0000
		σ_{t-1}^2	β_1	0.654	0.0000
		$u_{t-1}^2 I_{t-1}$	γ	0.601	0.0000
E-GARCH	$ln(\sigma_t^2) = a_0 + \beta_1 ln\sigma_{t-1}^2 + \gamma \frac{u_{t-1}}{\sigma_{t-1}} + \theta \frac{\lvert u_{t-1} \rvert}{\sigma_{t-1}}$	Constant	α_0	-1.093	0.0000
		$ln\sigma_{t-1}^2$	β_1	0.894	0.0000
		$\frac{u_{t-1}}{\sigma_{t-1}}$	γ	-0.281	0.0000
		$\frac{\lvert u_{t-1} \rvert}{\sigma_{t-1}}$	θ	0.097	0.0815

Interpretation of the GARCH Estimation

The α_0 is the intercept of the conditional variance equation of the GARCH specification. It is small and significant (because its p-value is less than all significance levels meaning it is less than 1%, 5%, and 10%).

The α_1 is the coefficient of the squared residuals of the conditional variance equation. It is around 0.23 and significant. Because its p-value is almost zero; less than all significance levels of 1%, 5%, and 10%. This parameter shows the reaction of conditional volatility to market shocks. The significance of this parameter confirms that the UK market volatility is quite sensitive to the shocks of the market. This is supported by many studies in the literature (Engle 1982; Bollerslev 1986; Akgiray 1989; West and Cho 1995; Chu et al. 2017; Costa 2017; Bhowmik and Wang 2020).

The β_1 is the coefficient of the GARCH term of the conditional variance equation. It is 0.72, a big number and since the p-value of this coefficient is also almost zero, less than all significance levels of 1%, 5%, and 10%, it is also significant. This parameter shows the persistence of the volatility suggesting strong evidence that the volatility of the FTSE 100 index over the estimation sub-period is quite persistent. This is supported by the previous works (Engle and Bollerslev 1986; McCurdy and Morgan 1988; Baillie and Bollerslev 1989; Hsieh 1989; Andersen and Bollerslev 1998; Joshi 2010; Li and Wang 2013; Francq and Zakoian 2019; Bhowmik and Wang 2020).

Another implication of the GARCH estimation from Table 1 is that although the conditional volatility of the FTSE 100 is persistent over the estimation period, it is still mean-reverting meaning that it will revert to its long-run mean variance. According to John et al. (2019), mean reversion means that current information does not influence the long-run forecast of the volatility in stationary GARCH-type models, and the volatility will revert to its long-run level, at a rate given by the sum of α_1 and β_1, which is usually close to one for financial time series. The sum of the two parameters is a proxy for the rate of convergence of conditional variance to its unconditional average.

From Table 1, the $\boldsymbol{\alpha_1} + \boldsymbol{\beta_1} = 0.227 + 0.723 = 0.95$ is less than one showing that although the GARCH process is persistent, it is not infinite, and the conditional variance of the market will converge to the unconditional variance at the rate of 0.95, the mean-reverting rate. The unconditional volatility could be calculated using Equation (7).

$$\bar{\sigma}^2 = \frac{\alpha_0}{1-(\alpha_1+\beta_1)} \tag{7}$$

$$\bar{\sigma}^2 = \frac{0.00000618}{1-(0.227+0.723)} = 0.000124$$

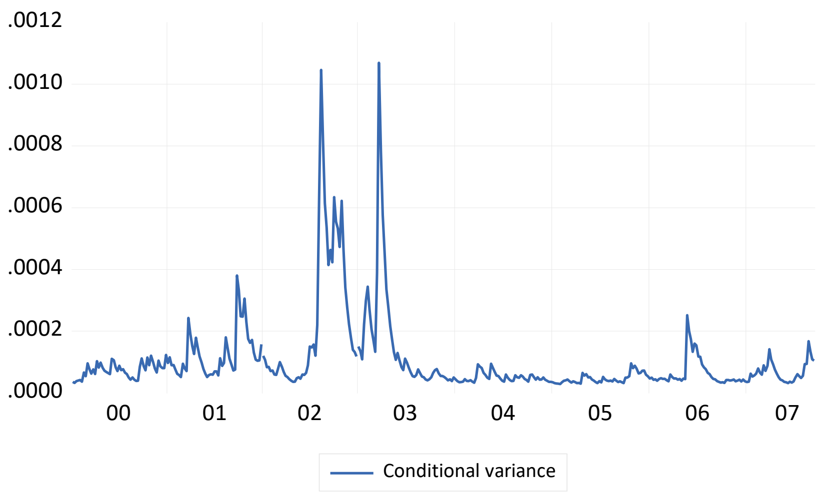
The average number of periods for the volatility to revert to its unconditional long-run level (0.000124) is measured by the half-life of the volatility shock. According to John et al. (2019), the half-life of the volatility shock is given by Equation (8).

$$\tau = \frac{log\frac{(\alpha_1+\beta_1)}{2}}{\log(\alpha_1+\beta_1)} \tag{8}$$

$$\tau = \frac{log\frac{(0.227+0.723)}{2}}{log(0.227+0.723)} = \frac{-0.32}{-0.022} = 14.5$$

Overall, For the FTSE 100, although the volatility of the returns is persistent over time, it is still mean-reverting. This finding is in agreement with Engle and Patton (2007) and John et al. (2019). Chart 3 visualises the conditional variance of the FTSE 100 using the GARCH model over the estimation sub-period. There is strong evidence of volatility clustering in the graph especially around the years 2002 and 2003. As the graph shows, in those times the conditional variance of the market exceeds 0.0010 but most of the time it is spared over its long term average which is 0.000124 based on equation (7).

Chart 3: Conditional Variance of the FTSE 100 using GARCH over the Estimation Period



Interpretation of the GJR-GARCH Estimation

From Table 1, the intercept, and the coefficient of the ARCH and GARCH terms, α_0, α_1 and β_1 are significant because their p-value is almost zero, less than all levels of 1%, 5%, and 10%. This is again confirming that the GARCH family can model the volatility of the UK stock market.

The α_1 and β_1 are -0.11 and 0.65 respectively and their significance again confirms the sensitivity of the UK market volatility to the shocks and persistency of the volatility. These agree with the results of the GARCH model and previous empirical studies. However, compared to the simple first-generation GARCH model, the GJR includes an extra term, $\boldsymbol{\gamma u_{t-1}^2 I_{t-1}}$, to be able to account for the asymmetries of the volatility of the markets. The γ is the asymmetry coefficient. This coefficient for the GJR estimation is 0.60 (see Table 1). Because its associated p-value is again almost zero, γ is significant at all levels showing a strong effect of asymmetry in the FTSE 100 volatility. This finding is supported by many studies (Olowe 2009; Chang et al. 2011; Abdalla and Suliman 2012; Hou 2013; Okicic 2014; Banumathy and Azhagaiah 2015; Bhowmik and Wang 2020).

Interpretation of the Results of the E-GARCH Estimation

The E-GARCH is another asymmetric volatility model and like the GJR-GARCH, is a way of parameterising the idea that in addition to the magnitude of the innovations (shocks), the sign of them could also influence the volatility. From Table 1, like the two previous models, the constant, the GARCH, and the ARCH terms, respectively α_0, β_1 and θ are significant (θ is only significant at 10%) implying that the GARCH models can capture the volatility features of the UK stock market.

The asymmetry term in the E-GARCH is defined as $\boldsymbol{\gamma \frac{u_{t-1}}{\sigma_{t-1}}}$. Based on Table 1, the $\boldsymbol{\gamma}$ is -0.28 and significant (because its p-value is 0) implying that the $\boldsymbol{\gamma}$ is significantly different from zero. This again proves the presence of asymmetry effect in the UK stock market. In addition to the fact that the asymmetry term is significantly different from zero, it is less than zero (negative amount) confirming the theory that the impact of negative shocks on the volatility of the UK stock market is higher than the impact of positive shocks of the same magnitude.

In the next step, the three models are compared, and the best-fitting model is chosen. (see Table 2)

Table 2: The In-Sample Performance of the Models over the Estimation Sub-period

Criteria	GARCH	GJR-GARCH	E-GARCH
Log-Likelihood	1327.59	1345.90	1347.12
Akaike Information Criterion	-6.651221	-6.738213	-6.744299
Schwarz Criterion	-6.611156	-6.688132	-6.694218
Hannan-Quinn Criterion	-6.635352	-6.718376	-6.724463

Based on the all in-sample evaluation criteria, the asymmetric GARCH models outpertform the standard GARCH, and the E-GARCH performs best among the three examined models. The best-fitting model is the model with the highest value of log-likelihood and the lowest values of the Information Criteria (AIC, SBC, HQC). The main reason for the approved superiority of the E-GARCH is that it is a non-linear and asymmetric version of the GARCH and can accommodate for the asymmetries of volatility. This finding is supported by many previous studies. The findings of this section could be summed up as follows.

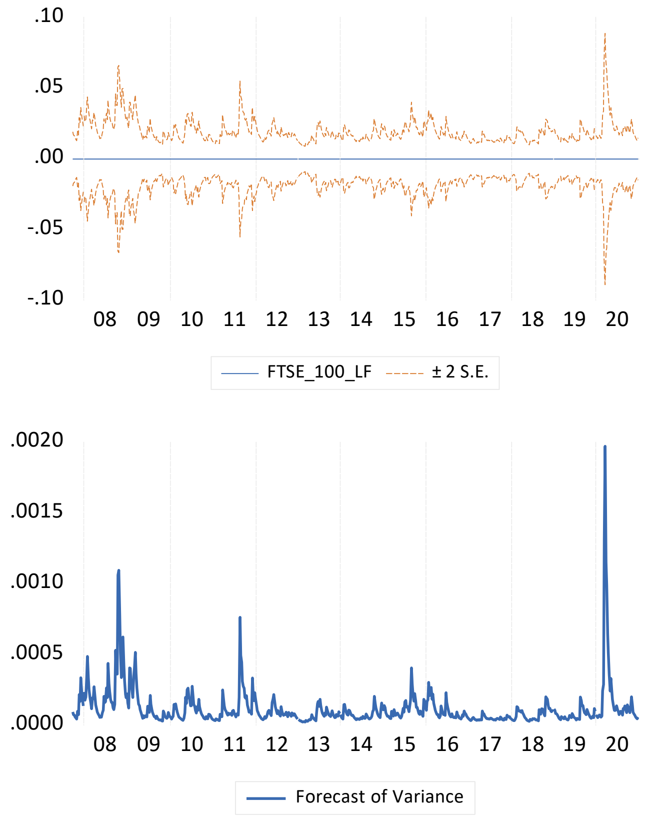
- The coefficients of the conditional variance of all three models are statistically significant confirming that all GARCH models can fit and estimate the volatility of the UK stock market. This is in agreement with Bhowmik and Wang (2020) and contradiction with Åstrand (2020).
- The volatility is appeared to be persistent and sensitive to shocks supported by Aliyev et al. (2020), and it is mean-reverting supported by John et al. (2019).
- The asymmetry terms incorporated in the GJR-GARCH and E-GARCH are also statistically significant (see Table 1), showing evidence of a strong leverage effect as supported by both the literature and the theory. It implies that the impacts of negative shocks on volatility are higher than those of positive shocks of the same magnitude (see e.g., (Okicic 2014; Banumathy and Azhagaiah 2015; Aliyev et al. 2020)).
- The non-linear asymmetric models perform better than the simple GARCH (see Table 2). It is supported by (e.g., Roni et al. 2017; Lin 2018; Dixit and Agrawal 2019; Hu 2019; Åstrand 2020). However, it is not supported by several studies (e.g., Hou 2013; Sharma 2015; Obeng 2016; Costa 2017; Bhowmik and Wang 2019).

- The E-GARCH is the best model for estimation purposes after being assessed by all 4 in-sample evaluation criteria. The superiority of exponential GARCH has been confirmed in several studies (e.g., Liu and Hung (2010); Gabriel (2012); and Lin (2018)).

4.2.2. Evaluation of the Out-of-sample Performance of the Models

The results of Section 4.2.1 showed that the E-GARCH model is the best-fitting model. To completely address objectives 1 and 2 of the research, it is needed to evaluate the out-of-sample performance of the models and examine if the best-fitting model is also the best one for the forecasting purpose. The Forecasting sub-Period is chosen from the mid-September of 2007 to the end of 2020. The reason is to examine if the best-fitting model is also the best in forecasting, especially when it comes to times of crisis. To perform the evaluation, firstly one-week-ahead forecasting is conducted. The results are shown in Figure 5.

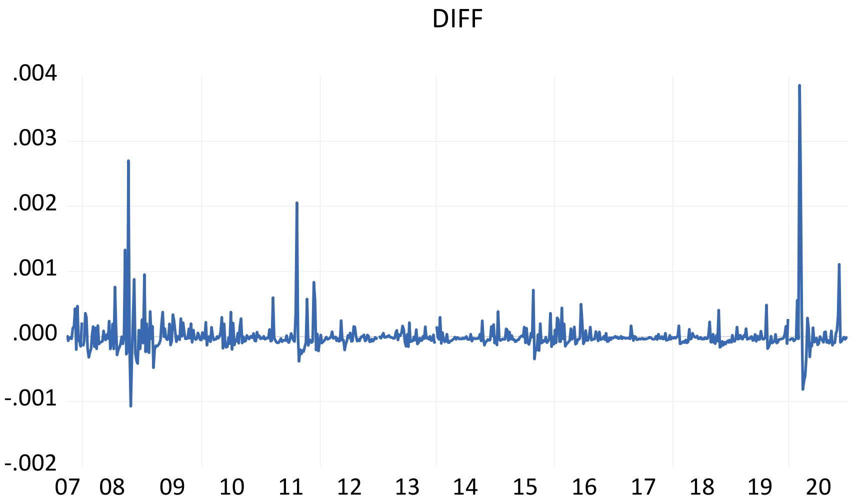
Figure 5: The One-Week-Ahead Forecast of the E-GARCH



Forecast: FTSE_100_LF	
Actual: FTSE_100_LOG_RETURNS	
Forecast sample: 9/19/2007 12/30/2020	
Included observations: 687	
Root Mean Squared Error	0.010957
Mean Absolute Error	0.007711
Mean Abs. Percent Error	121.4745
Theil Inequality Coef.	0.965868
Bias Proportion	0.001468
Variance Proportion	0.998525
Covariance Proportion	0.000007
Theil U2 Coefficient	0.991597
Symmetric MAPE	181.3183

The first criterion for out-of-sample performance evaluation is that the one-week-ahead forecasts of the E-GARCH model are compared with the realised variance. The proxy for the realized volatility is the measure of Equation (5) of this research. The result shows that the difference between the realised variance and the predicted variance is approximately zero except for the periods of the crisis (see Chart 4)

Chart 4: Difference between the Realised and Predicted Volatility using E-GARCH



The next step is to run statistical hypotheses to see if the predicted values for variance and the realised values are significantly different from zero.

To examine this, the realised weekly volatility values are regressed against the one-week-ahead forecasted values by conducting a series of Mincer-Zarnowitz regressions (see Equation (6)). Then two statistical hypothesis tests have been conducted. The first null hypothesis is that the slope coefficient of the regression is equal to one, and the alternative hypothesis is that the slope coefficient is different from one. This is to see if the forecasted variable and the realised variable are moving together one by one. The second null hypothesis is that the intercept coefficient is equal to zero and its alternative hypothesis is that the intercept coefficient is different from zero. The results of the tests show that the differences between the real data and the predicted data are not significantly different from zero and the predicted values using the E-GARCH are good predictors of the volatility of the FTSE 100. Figure 6 shows the results of the statistical regression and Table 2 summarise the results of the defined hypothesis tests.

Figure 6: The Regression of Realised against the Forecast Volatility using E-GARCH

Dependent Variable: VOLATILITY
Method: Least Squares
Date: 08/11/21 Time: 17:52
Sample: 9/19/2007 12/30/2020
Included observations: 687

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-5.50E-06	1.37E-05	-0.401565	0.6881
FORECAST	1.071854	0.072909	14.70125	0.0000

R-squared	0.239841	Mean dependent var	0.000120
Adjusted R-squared	0.238731	S.D. dependent var	0.000322
S.E. of regression	0.000281	Akaike info criterion	-13.51388
Sum squared resid	5.41E-05	Schwarz criterion	-13.50069
Log likelihood	4644.018	Hannan-Quinn criter.	-13.50878
F-statistic	216.1268	Durbin-Watson stat	1.586959
Prob(F-statistic)	0.000000		

Table 3: The summary of Statistical Hypothesis Tests

	Test 1	**Test 2**
	Slope Coefficient	Intercept Coefficient
Coefficient	1.071854	-0.0000055
Standard Error	0.072909	0.0000137
Null Hypothesis	$\beta = 1$	$\alpha = 0$
Alternative Hypothesis	$\beta \neq 1$	$\alpha \neq 0$
t-statistic	$\frac{\beta - 1}{Standard\ Error} = 0.986$	$\frac{\alpha - 0}{Standard\ Error} = -0.401$
Critical values from t-student table for a two-tail distribution for t-2 observations equal to 685 at 1%, 5%, and 10% levels	respectively 2.626, 1.985, and 1.660	
Decision about the null hypothesis	Cannot reject the Null	Cannot reject the Null
Result	The slope of the regression is not different from 1	The intercept of the regression is not different from 0
Interpretation	The E-GARCH model has a good predictivity power in forecasting volatility	The E-GARCH model has a good predictivity power in forecasting volatility

The results of the same process for the two other models (see Appendix D) shows that the GJR-GARCH and the simple classic GARCH do not exhibit good forecasting performance in terms of out-of-sample forecasting. To conclude the evaluation of the models using this criterion it can be claimed that the good in-sample performance of only the E-GARCH model can be translated to the good out-of-sample performance. This model was the best-fitting model in terms of in-sample estimation and comparing with the real-world data, it is the only model among the examined models of this research that can forecast accurately on an out-of-sample basis.

Table 4 is constructed for comparison among the models. However, using the values of loss functions, the E-GARCH is not ranked the best meaning that although the good in-sample performance of the GARCH may translate into its good out-of-sample performance within the Mincer-Zarnowitz series framework, using the loss functions as evaluation criteria does not confirm the superiority of the GARCH over the other two models. The loss functions including The RME, MAE, MAPE, and symmetric MAPE are error measuring criteria evaluating the forecasting models purely statistically and the lower these are, the better the models' forecast.

Table 4: The Out-of-Sample Performance of Models over the Forecasting Sub-period

Criteria	GARCH	GJR-GARCH	E-GARCH
Difference between the forecasted and the realised Volatility	Significantly different from zero	Significantly different from zero	Statistically zero
Root Mean Squared Error	0.01095200	0.01095000	0.01095700
Mean Absolute Error	0.00765600	0.00768500	0.00771100
Mean Absolute Percent Error	112.32	106.09	121.47
Symmetric MAPE	179.89	190.98	181.32

The findings of this section could be summarised as follows:

- Conducting a series of Mincer-Zarnowitz and comparing the performance of the models in forecasting the volatility of the UK market indicates that only the E-GARCH have been able to forecast the volatility of the FTSE 100 over mid-September of 2007 to the end of 2019 so that the difference between the realised volatility and the predicted volatility is statistically zero, suggesting a good predictive power of the E-GARCH.
- The E-GARCH is the only model performing well in forecasting after evaluation within an M-Z framework that compares the value of the realised volatility with the predicted one. However, utilising the loss functions, the E-GARCH is not ranked the best among the models. This is in agreement with Sharma (2015) claiming that

empirical results are dependent on the selection of performance evaluation criteria and different market conditions.

- Only the E-GARCH model performs accurately in terms of both in-sample and out-of-sample.
- Only the performance of the E-GARCH model to predict the volatility of the market in times of crisis is acceptable. It is reasonable that the behaviour of Volatility during crises is changing. According to Joldes (2019), the downward movements of the market index during the crisis increase the volatility (risk) of the market. Not only does the volatility of individual assets in distressed markets increase, but the way the returns of assets covary altogether is also changing in times of adverse events and crises (Hull 2012). That is why the performance of the GARCH models in normal markets is different from that of a distressed market. This alteration has been supported by several studies (amongst others (Franses and Van Dijk 1996; Granger and Poon 2001; Sharma 2015)).

So far, the two first objectives of the research have been addressed. To address the third objective, another sub-period has been chosen which excludes the crises from the time series. This is to examine if the market and economy's conditions alter the performance of the GARCH-family models. The next section explains it in more detail.

4.2.3. Estimations for the normal condition of the market, 2013-2018

To examine if the market and economy's conditions alter the performance of the GARCH-family models, a different sub-period has been chosen as a proxy for the normal market. This period contains the FTSE 100 returns from the beginning of 2013 to the end of 2018. The three selected models are run over this period and their estimation performance has been evaluated using 4 criteria (see Table 5). For detailed results see Appendix E.

Table 5: The In-Sample Performance of the Models over the Normal Market

Criteria	GARCH	GJR-GARCH	E-GARCH
Log-Likelihood	1070.24	1076.54	1078.13
Akaike Information Criterion	-6.901247	-6.935556	-6.945835
Schwarz Criterion	-6.852919	-6.875146	-6.885425
Hannan-Quinn Criterion	-6.881926	-6.911403	-6.921683

The findings of this section are listed as follows.

- Different markets show different volatility dynamics and best-fitting models could be chosen firstly considering market type and condition (Granger and Poon 2001; Hansen and Lunde 2005; Bhowmik and Wang 2020).
- The condition of the market alters the performance of the models. All information criteria exhibit the performance of the models has improved excluding the crisis.
- The E-GARCH is the best-fitting model during the normal conditions.
- Asymmetric models better fit the volatility of the UK market in normal conditions.
- The criteria matter. The log-likelihood function does not confirm the improvement of the models excluding the crisis.

Furthermore, in an attempt to make recommendations for the improvement of the models, firstly, an exogenous variable called VIX has been incorporated into the models and secondly, a different density function (t-student function) has been employed to examine if these can improve the ability of the models. The next section explains these tests in detail.

4.2.4. Improvement of the models

To address the last objective of the research, presenting recommendations for the improvement of the models, according to (Kambouroudis and McMillan 2016), the following approach is taken. Firstly, an exogenous variable called the the (VIX) has been incorporated into the variance equation of the models. The VIX is market participants' expectations about future volatility and some scholars believe that it carries incremental information above the GARCH specification (John et al. 2019).

After the inclusion of the VIX, the estimations have been conducted and the in-sample performance of the models has been compared with the baseline models (estimated in Section 4.2.3) to examine if the inclusion of an exogenous variable could improve the estimation performance of the models. Secondly, a different distribution function (t-student) has been employed and the results under this distribution function is compared with the baseline models. Finally, the inclusion of the VIX and employment of the t-student density function are jointly tested and compared with the baseline models. In this section, two categories of studies have been followed.

The first group of studies assert that the volatility of stock markets in addition to being a function of the squared residuals and the lagged volatility of the time-series itself, could be a function of some exogenous financial and economic variables (Engle and Patton 2007; Kambouroudis and McMillan 2016). As a result of the inclusion of the VIX index as an independent variable into all three selected models during the normal market condition, the performance of the models has improved using all evaluation criteria. The value of log-likelihood has increased and the values of the three information criteria (AIC, SBC, and HQC) has decreased as a result of the inclusion of the VIX for all models showing the improvement of the estimation performance of all models. For detailed results of the test see Appendix F. Descriptive statistics of the new independent variable (VIX) and its stationarity and Lagrange Multiplier tests confirming that the VIX is stationary and has a GARCH effect are also presented in Appendix F.

The second group of studies believe that the normal distribution function employed in the GARCH specification cannot completely capture the dynamics of the volatility of stock markets and employ fat-tail density function like t-student to improve the GARCH specification (Alberg et al. 2008; Wang et al. 2020). Contrary to these studies, performing the models of this research using the t-student density function cannot improve the performance of the models significantly. In some cases, it is observed that there are marginal improvements, but it is not consistent and meaningful (see Appendix G).

The joint inclusion of the VIX and the t-student, are also not appeared to outperform the performance of the models that solely include the VIX. However, it outperforms the baseline models (see Appendix H).

For ease of comparison, the results of the tests are summarised in Table 6.

Table 6: Comparison of Performance of the Models under Different Scenarios

	GARCH				GJR-GARCH				E-GARCH			
Criteria	**Baseline**	**VIX**	**T-student**	**Jointly VIX and t-student**	**Baseline**	**VIX**	**T-student**	**Jointly VIX and t-student**	**Baseline**	**VIX**	**T-student**	**Jointly VIX and t-student**
Log-Likelihood	1070.24	1090.31	1072.92	1088.48	1076.54	1091.79	1078.66	1092.49	1078.13	1100.41	1079.69	1100.36
Akaike Information Criterion	-6.901247	-7.024695	-6.912111	-7.006323	-6.935556	-7.027737	-0.942766	-7.025845	-6.945835	-7.083543	-6.949476	-7.076770
Schwarz Criterion	-6.852919	-6.964285	-6.851701	-6.933831	-6.875146	-6.955245	-6.870274	-6.941271	-6.885425	-7.011051	-6.876984	-6.992196
Hannan-Quinn Criterion	-6.881926	-7.000543	-6.887959	-6.977341	-6.911403	-6.998754	-6.913784	-6.992032	-6.921683	-7.054560	-6.920940	-7.042957

According to Table 6, the findings are summarised as follows.

- Inclusion of the implied volatility index to all models improves their performance compared to the baseline models. This finding is in agreement with previous studies amongst others (Kanniainen et al. 2014; Kambouroudis and McMillan 2016; Pati et al. 2018). The economic implication of this finding is that the VIX contains information helping the volatility modelling of the FTSE 100 that is not captured by the baseline models' specification. Theoretically, it is reasonable because VIX is defined as a proxy of expected market volatility and provides a forward-looking measure of the volatility of the stock markets.
- Employing t-student error distribution specification does not have a meaningful effect on the performance of the models. This finding is supported by Liu and Hung (2010) and Lin (2018) asserting that incorporating asymmetry components to the model is more important than error distribution specification. However, it is inconsistent with the studies conducted by the works implying that using a heavier tailed error distribution rather than a normal distribution significantly improves the models, and the asymmetric GARCH models under skewed Student-t distribution better characterize the pattern of volatility (Alberg et al. 2008; Wennström 2014; Wang et al. 2020).

The results of Section 4.2 strongly suggests that the asymmetric models better capture the dynamics of the UK stock market and the E-GARCH model is the best model among the examined models in terms of both estimation and forecasting performance. The final section of this chapter examines the adequacy of the E-GARCH model based on the dominant diagnostic checks in the literature.

4.3. Diagnostic Checks

According to the Box-Jenkins approach, this research uses an iterative process of identification, estimation, and diagnostic checks. So far, the appropriate models have been identified based on the common properties of the time-series of the FTSE 100, the estimation has been done using the selected models, the performance of the models has been evaluated using the determined criteria, and the most powerful model has been selected. The evidence of the research strongly suggests that the E-GARCH is the best model among the examined models based on all in-sample and out-of-sample criteria. The final step of the Box-Jenkins approach is to examine the adequacy of the selected model using common diagnostic checks. This section runs these tests to assess the adequacy of the E-GARCH in estimation and forecasting the volatility of the FTSE 100 index.

4.3.1. Ljung-Box Q-statistic

A common test for whether the E-GARCH model has adequately captured all the clustering, persistence, and autocorrelations in the volatility, is to examine the correlogram of the standardized squared residuals. For a volatility model to be adequate, the standardized squared residuals are required to be serially uncorrelated. (Engle and Patton 2007; John et al. 2019).

Figure 10 indicated the correlogram of standardised residuals of the E-GARCH models (the best-fitting model of the research) and their equivalent AC, PAC, and Q-statistics.

The Q-stats of Figure 10 is calculated using Ljung–Box Q-statistic developed by Ljung and Box (1978). The statistic follows the Chi-square distribution asymptotically with m degree of freedom and is specified as Equation (9).

$$Q^* = T(T+2)\sum_{K=1}^{m}\frac{\tau_k^2}{T-K} \tag{9}$$

$where\ T\ is\ the\ sample\ size, \tau\ is\ ACF, and\ m\ is\ the\ maximum\ lag\ length$

Figure 7: Correlogram of Standardised Residuals

Date: 08/29/21 Time: 07:43
Sample: 1/12/2000 9/12/2007
Included observations: 398

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.004	0.004	0.0079	0.929
		2	-0.032	-0.032	0.4192	0.811
		3	-0.061	-0.061	1.9056	0.592
		4	0.036	0.036	2.4339	0.657
		5	-0.039	-0.043	3.0467	0.693
		6	0.014	0.013	3.1228	0.793
		7	0.032	0.033	3.5274	0.832
		8	-0.012	-0.018	3.5877	0.892
		9	0.032	0.039	4.0119	0.911
		10	-0.031	-0.032	4.4097	0.927
		11	-0.016	-0.017	4.5167	0.952
		12	-0.014	-0.007	4.5945	0.970

*Probabilities may not be valid for this equation specification.

The null and alternative hypotheses are as follows:

$H_0: \tau_k = 0$

$H_1: \tau_k \neq 0$

The maximum examined lag length (m) is 12. All amounts of the Q-stat for all lags from figure 10 are lower than the critical values from the Chi-square distribution table at all significance levels of 1%, 5%, and 10% (see Figure 11). Therefore, the null hypothesis cannot be rejected. The non-rejection of the null hypothesis for the model implies that the autocorrelation functions for the lags up to a 12th lag, jointly together, are not significantly different from zero. This suggests that there is no evidence of autocorrelation among the residuals of the E-GARCH model. The model has captured all the autocorrelations which had initially been observed in the time series (see Section 4.1.4 of this research for the obvious evidence of initially observed autocorrelation in time series). This is supporting the adequacy of the E-GARCH in modelling and forecasting the dynamics of the volatility of the FTSE 100.

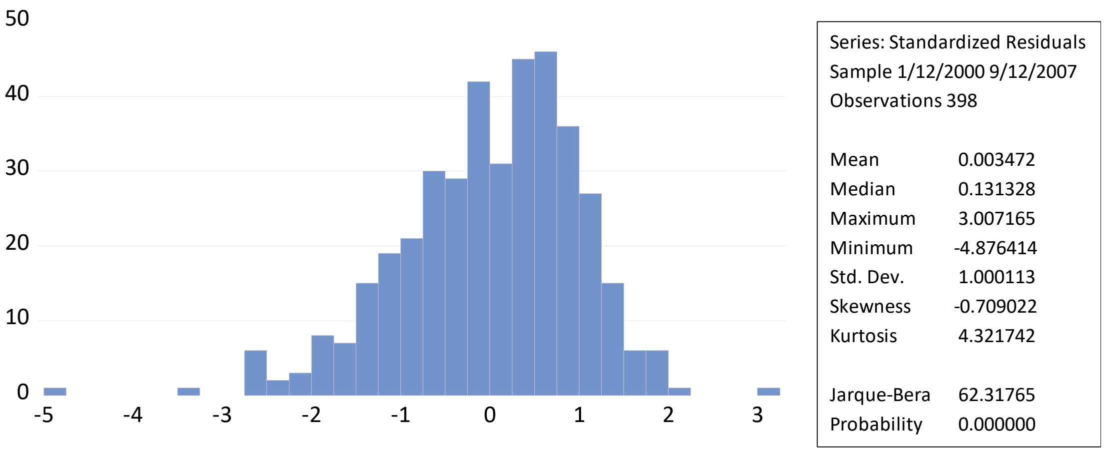
Figure 8: Critical Values from the Chi-Square Distribution Table

	Level of Significance α								
df	0.200	0.100	0.075	0.050	0.025	0.010	0.005	0.001	0.0005
1	1.642	2.706	3.170	3.841	5.024	6.635	7.879	10.828	12.116
2	3.219	4.605	5.181	5.991	7.378	9.210	10.597	13.816	15.202
3	4.642	6.251	6.905	7.815	9.348	11.345	12.838	16.266	17.731
4	5.989	7.779	8.496	9.488	11.143	13.277	14.860	18.467	19.998
5	7.289	9.236	10.008	11.070	12.833	15.086	16.750	20.516	22.106
6	8.558	10.645	11.466	12.592	14.449	16.812	18.548	22.458	24.104
7	9.803	12.017	12.883	14.067	16.013	18.475	20.278	24.322	26.019
8	11.030	13.362	14.270	15.507	17.535	20.090	21.955	26.125	27.869
9	12.242	14.684	15.631	16.919	19.023	21.666	23.589	27.878	29.667
10	13.442	15.987	16.971	18.307	20.483	23.209	25.188	29.589	31.421
11	14.631	17.275	18.294	19.675	21.920	24.725	26.757	31.265	33.138
12	15.812	18.549	19.602	21.026	23.337	26.217	28.300	32.910	34.822
13	16.985	19.812	20.897	22.362	24.736	27.688	29.820	34.529	36.479
14	18.151	21.064	22.180	23.685	26.119	29.141	31.319	36.124	38.111
15	19.311	22.307	23.452	24.996	27.488	30.578	32.801	37.698	39.720

4.3.2. Checking for the Normality of the Residuals

GARCH-family modelling uses the Maximum likelihood method in estimation. A commonly employed assumption while using the Maximum likelihood method is that the residuals of the model follow a normal distribution. The results of the Normality test for the standardised residuals of the model indicates that the residuals of the model are not following a normal distribution and one of the assumptions of the maximum likelihood method is violated (see Fique 9)

Figure 9: Histogram of the Standardised Residuals of the E-GARCH



From Figure 1, the test statistic of the Jarque-Bera normality test (Jarque and Bera 1987) is a huge number equal to 62.31 and From the Chi-squared distribution tables, the critical values at 1%, 5%, and 10% levels for 2 degrees of freedom are respectively 9.2, 5.99, and 4.605 (see Figure 8). Therefore, JB is so greater than the critical values and the null hypothesis that implies "$There\ is\ a\ normal\ distribution$" can be rejected, and the residuals are not normally distributed.

The histogram and the amount for the mean, median, maximum and minimum also confirm the result of the test. The skewness coefficient is less than one indicating that the returns distribution is negatively skewed. There is also a substantial excess kurtosis over 3. However, according to Bollerslev and Wooldridge (1992) even if the distribution of the residuals is not normal, the maximum likelihood estimates of the parameters of the GARCH model are consistent (Bollerslev and Wooldridge 1992; Engle and Patton 2007; John et al. 2019) so overall the E-GARCH adequately captures the dynamics of the volatility of the FTSE 100.

Chapter 5. Conclusion

5.1. Summary and Conclusion

This research evaluated the performance of three selected GARCH models including standard GARCH, and its two asymmetric extensions, the GJR-GARCH and Exponential-GARCH, in estimation and forecasting the weekly volatility of the FTSE 100 during 20 years of 2000-2020.

The conditional volatility of the FTSE 100 was found to be persistent, with a volatility half-life of 14.5 weeks, yet mean-reverting. The negative return innovations of the market were found to have an impact on conditional volatility more than positive return innovations of the same magnitude, confirming the asymmetry effect.

The empirical findings of the research revealed that the E-GARCH model which is an asymmetric extension of GARCH is the best fitting model in terms of in-sample performance confirming the idea supported by many previous works that heavily parameterized models which account for the asymmetries of the returns can better capture multiple dimensions of volatility dynamics.

Additionally, within an MZ regression framework and using different criteria, the evaluation of the performance of GARCH-family models by producing one-step-ahead volatility forecasts for the UK market revealed that the good estimation performance of the E-GARCH may translate into its good out-of-sample performance as well, and the E-GARCH outperformed the other two models both in estimation and forecasting.

After excluding the well-known crisis from the time series it was seen that the performance of the models significantly changed confirming that adverse events alter the ability of the GARCH models. During the normal market condition, the in-sample performance of the models improved.

The research also found evidence consistent with the theoretical result that the models perform better when VIX is included as an independent variable in the variance equation of the model. This finding confirms that the VIX contains incremental information over the GARCH specification. Hence, the inclusion of the VIX into the GARCH models is recommended to improve their performance.

However, the results were less clear-cut when the models were performed under a heavier fat distribution function (t-student). Employing the t-student density function did not appear meaningful incremental explanatory power over the baseline models. This implies that contrary to several studies, t-student did not have extra benefits that had not been captured by the GARCH models employing a normal distribution. In another word, although it intuitively seemed that the fatter tail functions are better able to capture the dynamics of volatility, the models with the inclusion of VIX encompassed the models with the heavier tail density functions. This is supported by Liu and Hung (2010) and Lin (2018).

5.2. Recommendations

Considering the findings of the research, the following points are recommended to improve the volatility modelling and forecasting process.

- Incorporating asymmetry components into the models significantly improves both the in-sample and out-of-sample performance of volatility models. The heavily parameterized non-linear models as many studies confirm are better able to capture different dimensions of volatility. For stock markets, it is highly unlikely that negative and positive shocks have the same effect on variance so employing the asymmetric GARCH is recommended for the FTSE 100 volatility estimation and forecasting.
- Incorporating the “Implied Volatility Index” of the FTSE 100 into the variance equation of the GARCH can significantly improve the performance of the GARCH models. This is also supported by previous studies and the logic behind it is that the VIX is a forward-looking volatility index and carries extra information content over the GARCH specification.
- Although it is well established that the unconditional distribution of asset returns fluctuations has heavy tails, based on the findings of this research the inclusion of the VIX is more important than error distribution specification and employing the fatter tailed density of t-student did not show improvement over the models employing a normal distribution and containing the VIX.
- Among the selected models, employing the E-GARCH is recommended since it outperformed the other two models both in estimation and forecasting the volatility of the UK stock market over 2000-2020. A good volatility model must be able to capture and reflect commonly held stylised facts about conditional volatility,

meaning unique characteristics of stock returns volatility such as leptokurtic distribution of returns, volatility clustering, asymmetric volatility, and mean-reverting volatility (Hussain et al. 2019). The exponential-GARCH which allows for the inclusion of asymmetric variables, better fitted the so-called stylised facts of the UK's FTSE 100 over 2000-2020 and consistently outperformed the other models.

Before using the findings of this research, the limitations of the work should be considered by the users of the research.

5.3. Limitations

One drawback of the GARCH models is that the empirical results of studies are dependent on the sampling frequency (Engle and Patton 2007; Francq and Zakoian 2019). This research has used weekly time series and using a different frequency could have resulted in different findings. Andersen and Bollerslev (1998) recommend that intraday high-frequency returns could better reflect the dynamics of stock market volatility. Using the high-frequency data (e.g., 5-minute data) was beyond the scope of this work.

Although this research has tried to appropriately fit the models into the UK market's data, special care should be taken in generalizing the results of this research into different datasets, different markets, and different economic conditions. Because a key challenge in GARCH modelling is finding the common characteristics in the stock market that fits the model (Bhowmik and Wang 2020).

Another limitation is that the way the sample data is split into in-sample and out-of-sample is relatively arbitrary. Different periods could have resulted in different findings. Moreover, this research has done single-period forecasting using one-week-ahead horizons. Multi-period forecasting could have resulted in different performances for the selected models.

Finally, the limitations of the information criteria used in the research for evaluation purposes should be considered. The AIC, SBC, and HQC are purely statistical measures and some authors argue that in some cases those criteria need modifications to be economically meaningful as well (Brooks and Burke 2003; Aliyev et al. 2020).

Considering the limitations of this research, there is an extended horizon for future research.

5.4. Scope for further research

- Utilising multi-horizon forecasting instead of one-step-ahead forecasting
- Using volatility models which allow for volatility breaks, such as the Markov–switching models of conditional heteroscedasticity (see e.g., Lange and Rahbek (2009)).
- Examining different sectoral indices of the UK stock market. That will give additional insight into the volatility spill-over from one market or sector to another one.
- The study can be extended more to model and compare the volatility of different developed and developing countries.
- Using the Superior predictive ability test (SPA) of Hansen for evaluation purposes.

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Appendices

Appendix A. Ethics Checklist

Bournemouth University

Research Ethics Checklist

About Your Checklist	
Ethics ID	39105
Date Created	09/07/2021 10:03:48
Status	Approved
Date Approved	15/07/2021 11:40:32
Date Submitted	14/07/2021 10:32:57
Risk	Low

Researcher Details	
Name	Leila Majidizavieh
Faculty	BU Business School
Status	Postgraduate Taught (Masters, MA, MSc, MBA, LLM)
Course	MSc Finance
Have you received funding to support this research project?	No

Project Details	
Title	**An Evaluation of Various Generalised Autoregressive Conditional Heteroskedasticity Models** *What makes a model superior in Estimation and Forecasting UK Stock Market's Volatility*
Start Date of Project	02/06/2021
End Date of Project	03/09/2021
Proposed Start Date of Data Collection	19/07/2021
Supervisor	Rubina Islam
Approver	Rubina Islam

Summary - no more than 600 words (including detail on background methodology, sample, outcomes, etc.)

This research aims to evaluate the performance of some selected GARCH-type models in estimation and forecasting weekly volatility of the UK stock market (FTSE-100 as a proxy) during 2006-2020 and tries to see if there are models that systematically outperform the other ones and if so, which conditions make some models superior over the other ones. The output of the research is recommendations around what makes a volatility estimation model a good model for forecasting purposes.

The objectives are:

- To evaluate the ability of standard GARCH, Exponential GARCH, and the GJR-GARCH in modelling and forecasting the weekly volatility of FTSE-100.

- To find if non-linear GARCH models can outperform the standard GARCH model and if so, to find the best-fitting model among two variants of the non-linear GARCH models.
- To compare the accuracy of models in the normal course of the economy with the crisis periods and examine if distressed markets' characteristics alter the performance of models.
- To explore possible sources of improvements in models to make recommendations that benefit both future empirical works and stock market participants.

Filter Question: Does your study involve the use or re-use of data which will be obtained from a source other than directly from a Research Participant?

Additional Details	
Please describe the data, its source and how you are permitted to use it	The data are time-series related to the UK stock market. Weakly closed values of the FTSE-100 index over the period 2006-2020 will be used to estimate the proposed model. The main source of data will be the Federal Reserve Economic Data (FRED) database and other free publically available websites including London Stock Exchange. FRED is a free database available on the internet. The EViews package will be used for estimation purposes.

Research Data	
Will identifiable personal information be collected, i.e. at an individualised level in a form that identifies or could enable identification of the participant?	No
Will research outputs include any identifiable personal information i.e. data at an individualised level in a form which identifies or could enable identification of the individual?	No

Storage, Access and Disposal of Research Data	
Where will your research data be stored and who will have access during and after the study has finished.	
My research data are publically available data. It will be downloaded in spreadsheets format and will be stored on my personal laptop.	
Once your project completes, will any anonymised research data be stored on BU's Online Research Data Repository "BORDaR"?	Yes

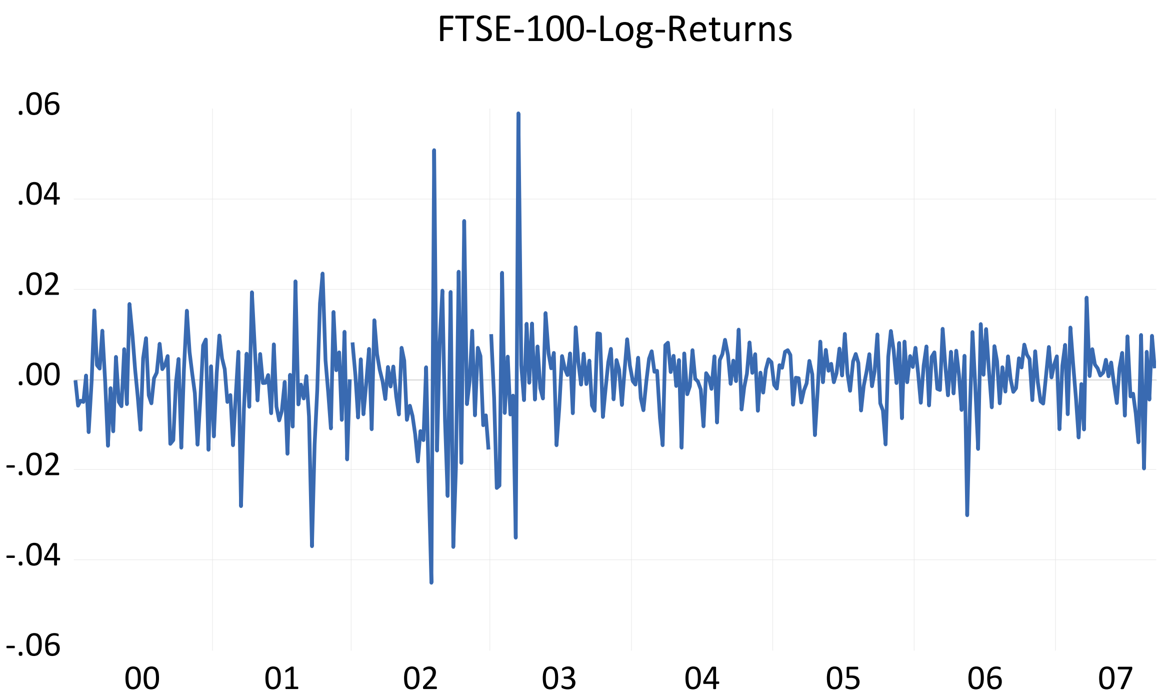
Final Review	
Are there any other ethical considerations relating to your project which have not been covered above?	No

Risk Assessment	
Have you undertaken an appropriate Risk Assessment?	Yes

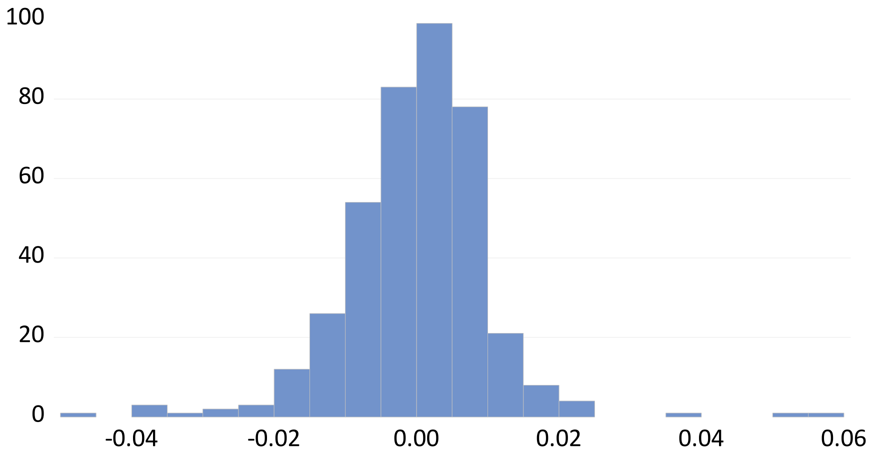
Appendix B. Tests for the Common Properties of the Time series

Appendix B1. Estimation Sub-period, 2000-2007

Weekly Log-Returns of the FTSE 100 over 2000-2007



Histogram and Descriptive Statistics of the FTSE 100 over 2000-2007



Series: FTSE_100_LOG_RETURNS
Sample 1/12/2000 9/12/2007
Observations 398

Mean	-3.90e-05
Median	0.000856
Maximum	0.059010
Minimum	-0.045117
Std. Dev.	0.010063
Skewness	0.155873
Kurtosis	9.138188
Jarque-Bera	626.4277
Probability	0.000000

Correlogram of the Returns of the FTSE 100 over 2000-2007

Date: 08/11/21 Time: 10:38
Sample: 1/12/2000 9/12/2007
Included observations: 398

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.155	-0.155	9.6744	0.002
		2	-0.013	-0.038	9.7422	0.008
		3	0.077	0.071	12.122	0.007
		4	-0.130	-0.110	18.931	0.001
		5	-0.011	-0.046	18.976	0.002
		6	0.090	0.076	22.302	0.001
		7	-0.095	-0.058	26.008	0.001
		8	-0.019	-0.054	26.158	0.001
		9	0.031	0.003	26.555	0.002
		10	-0.019	0.014	26.705	0.003
		11	0.103	0.096	31.048	0.001
		12	-0.022	-0.014	31.239	0.002

Stationarity Test (ADF Test) for the Returns of the FTSE 100 over 2000-2007

Null Hypothesis: FTSE_100_LOG_RETURNS has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=16)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-23.24241	0.0000
Test critical values:	1% level	-3.446608	
	5% level	-2.868601	
	10% level	-2.570597	

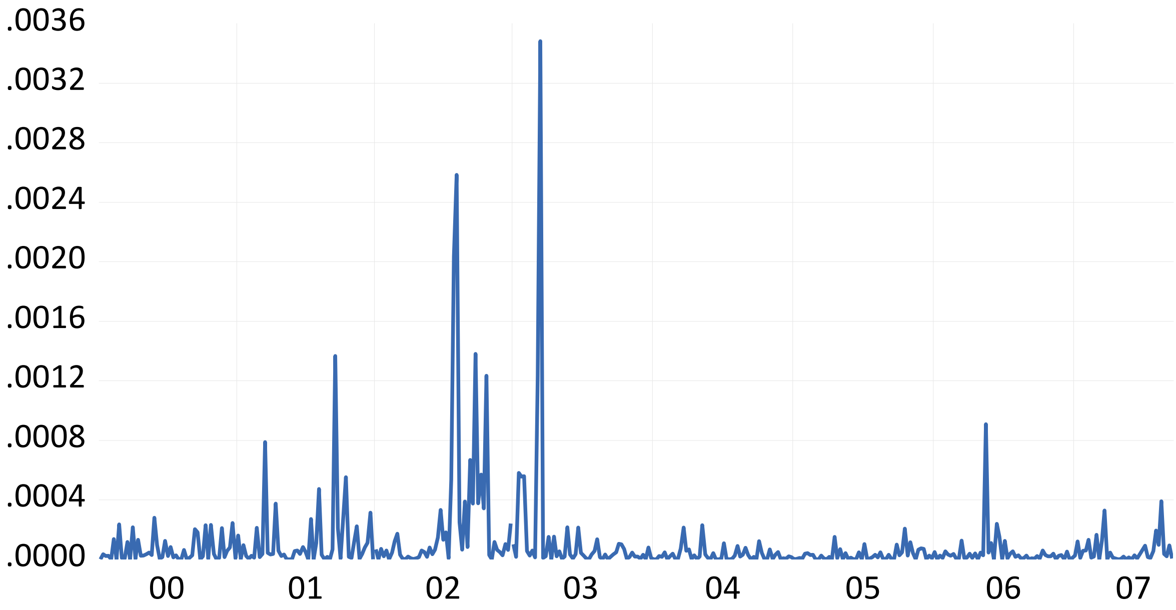
*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(FTSE_100_LOG_RETURNS)
Method: Least Squares
Date: 08/11/21 Time: 10:40
Sample (adjusted): 1/19/2000 9/12/2007
Included observations: 397 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
FTSE_100_LOG_RETURNS(-1)	-1.155347	0.049709	-23.24241	0.0000
C	-4.57E-05	0.000500	-0.091305	0.9273

R-squared	0.577635	Mean dependent var	6.69E-06
Adjusted R-squared	0.576566	S.D. dependent var	0.015315
S.E. of regression	0.009966	Akaike info criterion	-6.374350
Sum squared resid	0.039228	Schwarz criterion	-6.354279
Log likelihood	1267.308	Hannan-Quinn criter.	-6.366399
F-statistic	540.2097	Durbin-Watson stat	2.010619
Prob(F-statistic)	0.000000		

Squared Returns of the FTSE 100 over 2000-2007



Correlogram of the Squared Returns of the FTSE 100 over 2000-2007

Date: 08/11/21 Time: 10:50
Sample: 1/12/2000 9/12/2007
Included observations: 398

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.361	0.361	52.222	0.000
		2	0.097	-0.039	55.980	0.000
		3	0.074	0.059	58.164	0.000
		4	0.120	0.089	64.002	0.000
		5	0.104	0.034	68.383	0.000
		6	0.178	0.145	81.206	0.000
		7	0.200	0.099	97.500	0.000
		8	0.147	0.038	106.27	0.000
		9	0.072	-0.006	108.37	0.000
		10	0.076	0.029	110.76	0.000
		11	0.120	0.061	116.72	0.000
		12	0.073	-0.030	118.92	0.000

Heteroskedasticity Test (ARCH Effect) over 2000-2007

Heteroskedasticity Test: ARCH

F-statistic	58.89360	Prob. F(1,395)	0.0000
Obs*R-squared	51.51154	Prob. Chi-Square(1)	0.0000

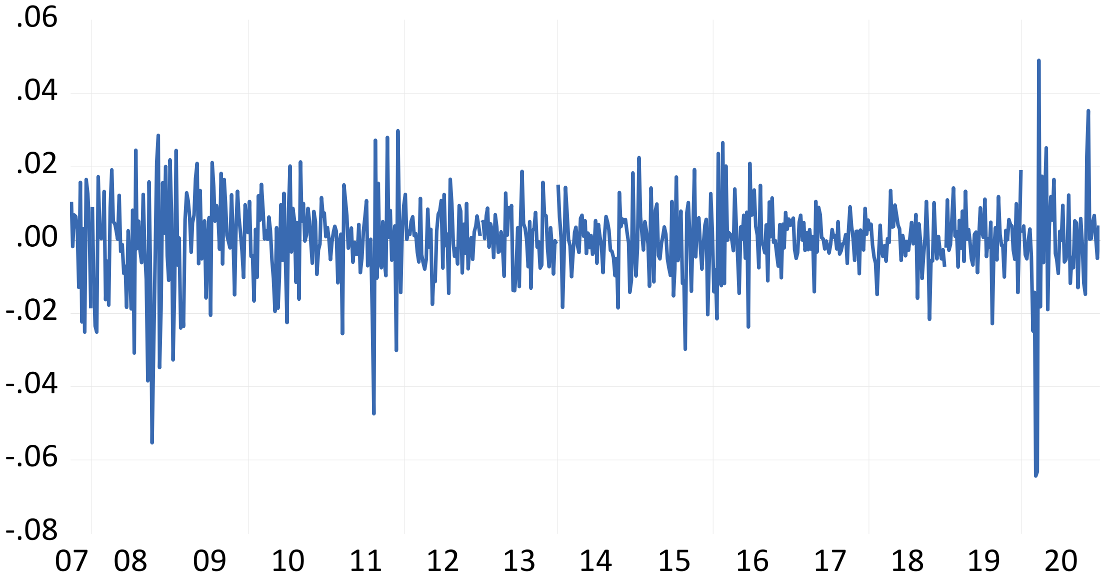
Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 08/11/21 Time: 11:00
Sample (adjusted): 1/19/2000 9/12/2007
Included observations: 397 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	6.48E-05	1.43E-05	4.515232	0.0000
RESID^2(-1)	0.360204	0.046937	7.674216	0.0000

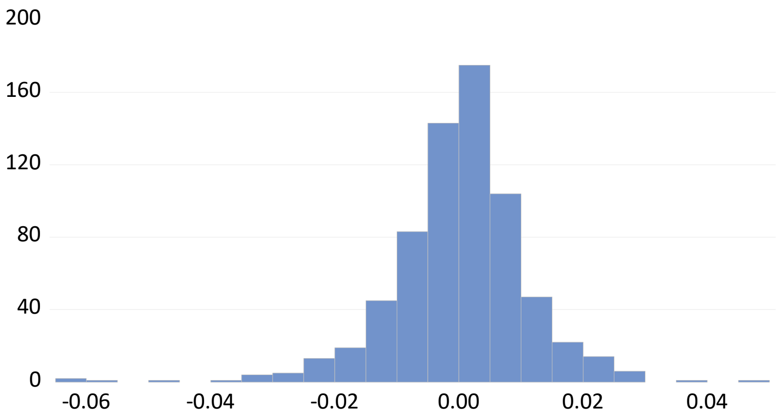
R-squared	0.129752	Mean dependent var	0.000101
Adjusted R-squared	0.127549	S.D. dependent var	0.000289
S.E. of regression	0.000270	Akaike info criterion	-13.59304
Sum squared resid	2.87E-05	Schwarz criterion	-13.57297
Log likelihood	2700.218	Hannan-Quinn criter.	-13.58509
F-statistic	58.89360	Durbin-Watson stat	1.972269
Prob(F-statistic)	0.000000		

Appendix B2. Forecasting Sub-period, 2007-2020

Weekly Log-Returns of the FTSE 100 over 2007-2020



Histogram and Descriptive Statistics of the FTSE 100 over 2000-2007



Series: FTSE_100_LOG_RETURNS
Sample 9/19/2007 12/30/2020
Observations 687

Mean	2.45e-05
Median	0.000581
Maximum	0.049062
Minimum	-0.064383
Std. Dev.	0.010957
Skewness	-0.877193
Kurtosis	8.205143
Jarque-Bera	863.6559
Probability	0.000000

Correlogram of the Returns of the FTSE 100 over 2007-2020

Date: 08/29/21 Time: 10:16
Sample: 9/19/2007 12/30/2020
Included observations: 687

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.022	-0.022	0.3362	0.562
		2	-0.041	-0.042	1.5087	0.470
		3	-0.028	-0.030	2.0576	0.561
		4	-0.011	-0.014	2.1434	0.709
		5	0.068	0.065	5.3710	0.372
		6	-0.079	-0.078	9.6996	0.138
		7	-0.075	-0.074	13.573	0.059
		8	0.075	0.070	17.450	0.026
		9	-0.015	-0.021	17.616	0.040
		10	-0.067	-0.075	20.726	0.023
		11	-0.080	-0.074	25.238	0.008
		12	-0.013	-0.017	25.363	0.013

Stationarity Test (ADF Test) for the Returns of the FTSE 100 over 2007-2020

Null Hypothesis: FTSE_100_LOG_RETURNS has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-26.75250	0.0000
Test critical values:	1% level	-3.439654	
	5% level	-2.865536	
	10% level	-2.568955	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(FTSE_100_LOG_RETURNS)
Method: Least Squares
Date: 08/29/21 Time: 10:19
Sample (adjusted): 9/26/2007 12/30/2020
Included observations: 686 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
FTSE_100_LOG_RETURNS(-1)	-1.022077	0.038205	-26.75250	0.0000
C	9.73E-06	0.000419	0.023250	0.9815
R-squared	0.511323	Mean dependent var	-9.43E-06	
Adjusted R-squared	0.510608	S.D. dependent var	0.015671	
S.E. of regression	0.010963	Akaike info criterion	-6.185725	
Sum squared resid	0.082204	Schwarz criterion	-6.172515	
Log likelihood	2123.704	Hannan-Quinn criter.	-6.180614	
F-statistic	715.6962	Durbin-Watson stat	2.001223	
Prob(F-statistic)	0.000000			

Heteroskedasticity Test (ARCH Effect) over 2000-2007

Heteroskedasticity Test: ARCH

F-statistic	213.5641	Prob. F(1,684)	0.0000
Obs*R-squared	163.2251	Prob. Chi-Square(1)	0.0000

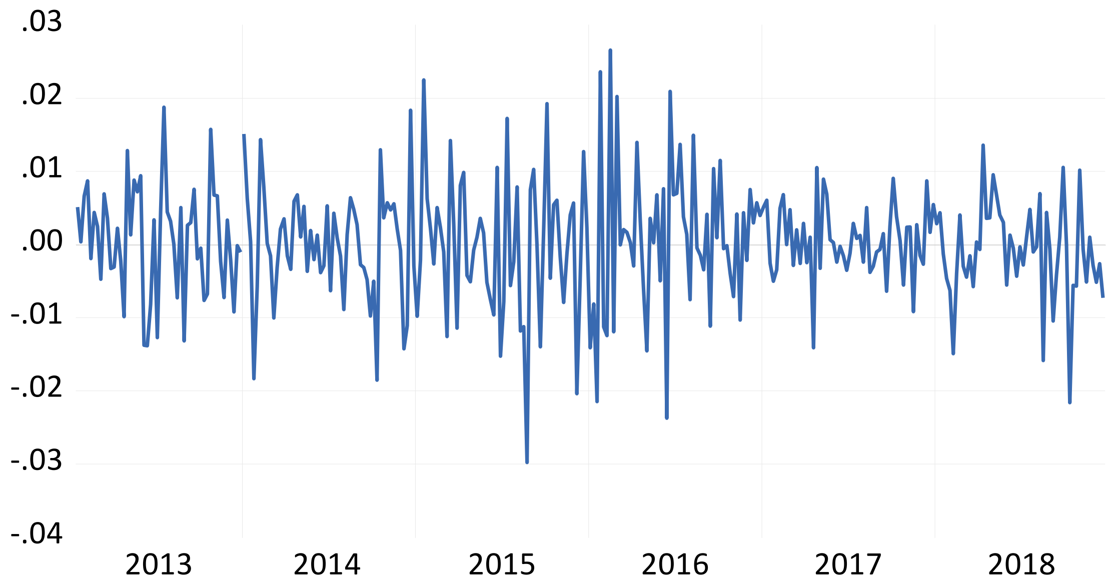
Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 08/29/21 Time: 10:45
Sample (adjusted): 9/26/2007 12/30/2020
Included observations: 686 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	6.13E-05	1.15E-05	5.347455	0.0000
RESID^2(-1)	0.487825	0.033381	14.61383	0.0000

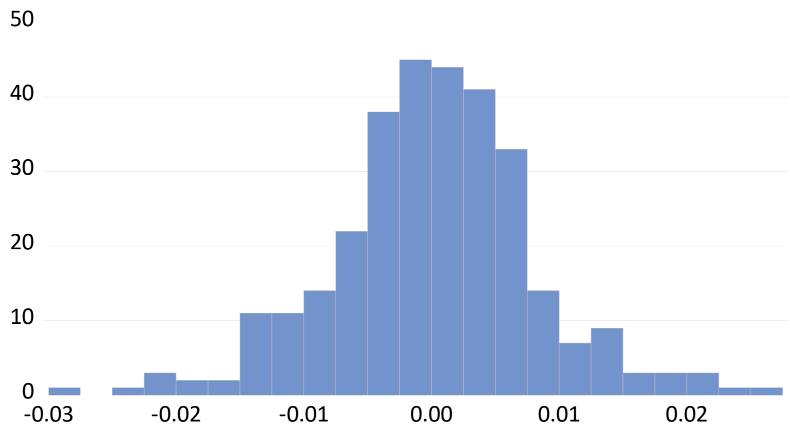
R-squared	0.237937	Mean dependent var	0.000120
Adjusted R-squared	0.236823	S.D. dependent var	0.000322
S.E. of regression	0.000282	Akaike info criterion	-13.50992
Sum squared resid	5.42E-05	Schwarz criterion	-13.49671
Log likelihood	4635.901	Hannan-Quinn criter.	-13.50481
F-statistic	213.5641	Durbin-Watson stat	1.999615
Prob(F-statistic)	0.000000		

Appendix B3. Normal Market Sub-period, 2013-2018

Weekly Log-Returns of FTSE 100 over 2013-2018



Histogram and Descriptive Statistics of the FTSE 100 over 2013-2018



Series: FTSE
Sample 1/09/2013 12/19/2018
Observations 309

Mean	0.000162
Median	0.000284
Maximum	0.026581
Minimum	-0.029771
Std. Dev.	0.008012
Skewness	-0.092126
Kurtosis	4.198192
Jarque-Bera	18.92128
Probability	0.000078

Correlogram of the Returns of the FTSE 100 over 2013-2018

Date: 08/29/21 Time: 10:55
Sample: 1/09/2013 12/19/2018
Included observations: 309

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.071	-0.071	1.5856	0.208
		2	-0.079	-0.084	3.5330	0.171
		3	-0.058	-0.071	4.6035	0.203
		4	-0.007	-0.024	4.6172	0.329
		5	0.008	-0.006	4.6351	0.462
		6	-0.110	-0.119	8.4570	0.206
		7	-0.062	-0.085	9.6675	0.208
		8	0.072	0.040	11.334	0.183
		9	-0.022	-0.043	11.491	0.244
		10	0.012	0.001	11.535	0.317
		11	0.076	0.079	13.408	0.268
		12	0.021	0.021	13.553	0.330

Stationarity Test (ADF Test) for the Returns of the FTSE 100 over 2013-2018

Null Hypothesis: FTSE has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=15)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-18.77656	0.0000
Test critical values:	1% level	-3.451421	
	5% level	-2.870712	
	10% level	-2.571728	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(FTSE)
Method: Least Squares
Date: 08/29/21 Time: 10:56
Sample (adjusted): 1/16/2013 12/19/2018
Included observations: 308 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
FTSE(-1)	-1.071483	0.057065	-18.77656	0.0000
C	0.000160	0.000457	0.349813	0.7267

R-squared	0.535349	Mean dependent var	-4.02E-05
Adjusted R-squared	0.533831	S.D. dependent var	0.011735
S.E. of regression	0.008012	Akaike info criterion	-6.809192
Sum squared resid	0.019645	Schwarz criterion	-6.784970
Log likelihood	1050.616	Hannan-Quinn criter.	-6.799507
F-statistic	352.5594	Durbin-Watson stat	2.009413
Prob(F-statistic)	0.000000		

Heteroskedasticity Test (ARCH Effect) over 2013-2018

Heteroskedasticity Test: ARCH

F-statistic	7.184271	Prob. F(1,306)	0.0078
Obs*R-squared	7.065347	Prob. Chi-Square(1)	0.0079

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 08/29/21 Time: 10:58
Sample (adjusted): 1/16/2013 12/19/2018
Included observations: 308 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	5.44E-05	7.42E-06	7.337523	0.0000
RESID^2(-1)	0.151430	0.056496	2.680349	0.0078

R-squared	0.022939	Mean dependent var	6.41E-05
Adjusted R-squared	0.019746	S.D. dependent var	0.000115
S.E. of regression	0.000114	Akaike info criterion	-15.32079
Sum squared resid	3.95E-06	Schwarz criterion	-15.29656
Log likelihood	2361.401	Hannan-Quinn criter.	-15.31110
F-statistic	7.184271	Durbin-Watson stat	2.051401
Prob(F-statistic)	0.007753		

Appendix C. Estimation

The Results from the GARCH over the Estimation Sub-period

Dependent Variable: FTSE_100_LOG_RETURNS
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 08/11/21 Time: 11:12
Sample: 1/12/2000 9/12/2007
Included observations: 398
Convergence achieved after 21 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000457	0.000422	1.082197	0.2792
		Variance Equation		
C	6.18E-06	1.98E-06	3.122579	0.0018
RESID(-1)^2	0.227741	0.049176	4.631160	0.0000
GARCH(-1)	0.723181	0.047431	15.24709	0.0000

R-squared	-0.002433	Mean dependent var	-3.90E-05
Adjusted R-squared	-0.002433	S.D. dependent var	0.010063
S.E. of regression	0.010075	Akaike info criterion	-6.651221
Sum squared resid	0.040296	Schwarz criterion	-6.611156
Log likelihood	1327.593	Hannan-Quinn criter.	-6.635352
Durbin-Watson stat	2.304883		

The Results from the GJR-GARCH over the Estimation Sub-period

Dependent Variable: FTSE_100_LOG_RETURNS
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 08/11/21 Time: 11:50
Sample: 1/12/2000 9/12/2007
Included observations: 398
Convergence achieved after 17 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-1)^2*(RESID(-1)<0)
+ C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000130	0.000411	-0.317020	0.7512
		Variance Equation		
C	1.34E-05	2.42E-06	5.536240	0.0000
RESID(-1)^2	-0.111158	0.024860	-4.471322	0.0000
RESID(-1)^2*(RESID(-1)<0)	0.601829	0.121697	4.945291	0.0000
GARCH(-1)	0.654312	0.059053	11.07999	0.0000

R-squared	-0.000082	Mean dependent var	-3.90E-05
Adjusted R-squared	-0.000082	S.D. dependent var	0.010063
S.E. of regression	0.010063	Akaike info criterion	-6.738213
Sum squared resid	0.040201	Schwarz criterion	-6.688132
Log likelihood	1345.904	Hannan-Quinn criter.	-6.718376
Durbin-Watson stat	2.310300		

The Results from the E-GARCH over the Estimation Sub-period

Dependent Variable: FTSE_100_LOG_RETURNS
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 08/11/21 Time: 11:54
Sample: 1/12/2000 9/12/2007
Included observations: 398
Convergence achieved after 32 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(2) + C(3)*ABS(RESID(-1)/@SQRT(GARCH(-1))) +
C(4)*RESID(-1)/@SQRT(GARCH(-1)) + C(5)*LOG(GARCH(-1))

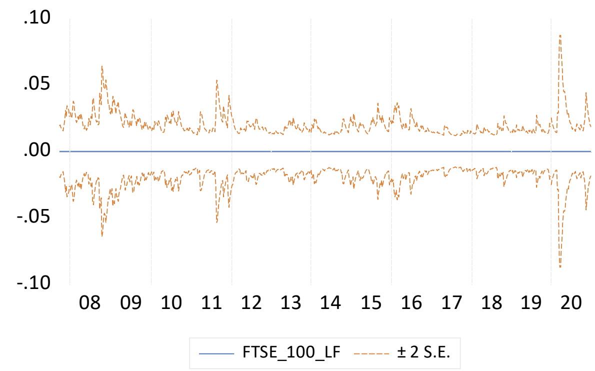
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000200	0.000419	-0.477183	0.6332
Variance Equation				
C(2)	-1.093658	0.206262	-5.302276	0.0000
C(3)	0.096899	0.055630	1.741836	0.0815
C(4)	-0.281342	0.042789	-6.575169	0.0000
C(5)	0.893694	0.020965	42.62817	0.0000

R-squared	-0.000257	Mean dependent var	-3.90E-05
Adjusted R-squared	-0.000257	S.D. dependent var	0.010063
S.E. of regression	0.010064	Akaike info criterion	-6.744299
Sum squared resid	0.040208	Schwarz criterion	-6.694218
Log likelihood	1347.116	Hannan-Quinn criter.	-6.724463
Durbin-Watson stat	2.309897		

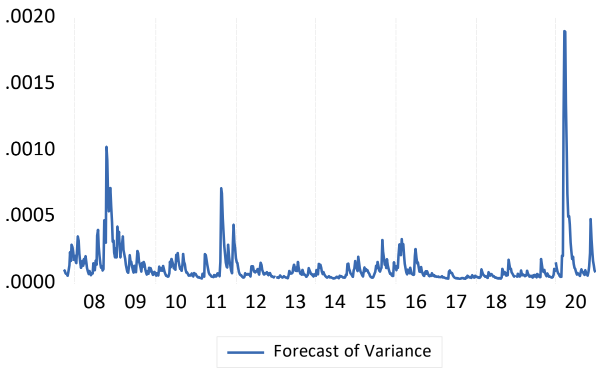
Appendix D. Forecasting

Appendix D1. Forecasting using the GARCH Model

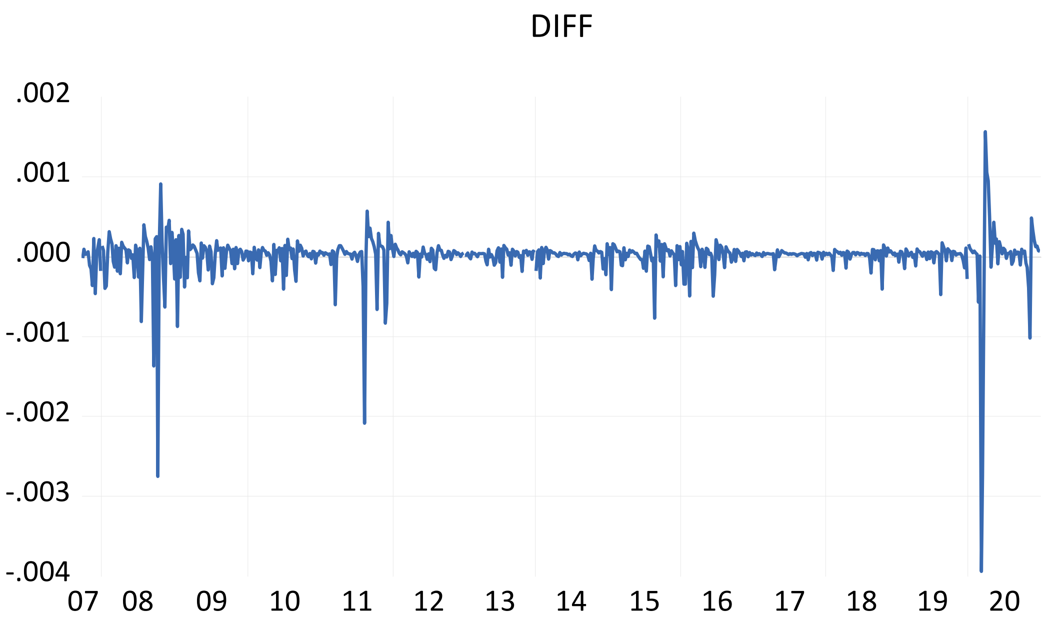
The One-Week-Ahead Forecast of the GARCH



Forecast: FTSE_100_LF	
Actual: FTSE_100_LOG_RETURNS	
Forecast sample: 9/19/2007 12/30/2020	
Included observations: 687	
Root Mean Squared Error	0.010952
Mean Absolute Error	0.007656
Mean Abs. Percent Error	112.3172
Theil Inequality Coef.	0.972839
Bias Proportion	0.000677
Variance Proportion	0.999319
Covariance Proportion	0.000005
Theil U2 Coefficient	1.005622
Symmetric MAPE	179.8916



The Difference between the Realised and Predicted Volatility using E-GARCH



The Regression of Realised against the Predicted Volatility using E-GARCH

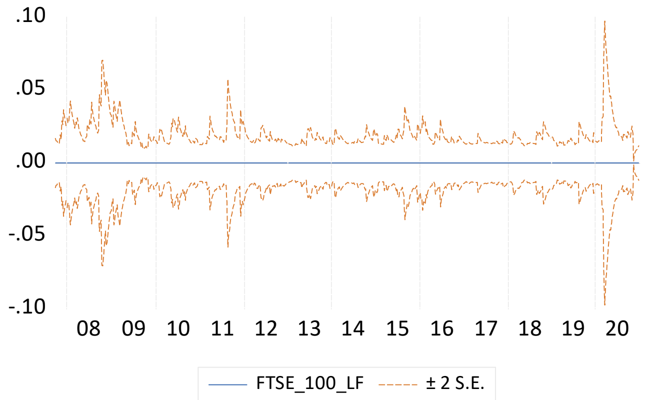
Dependent Variable: VOLATILITY
Method: Least Squares
Date: 08/11/21 Time: 17:24
Sample: 9/19/2007 12/30/2020
Included observations: 687

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.81E-05	1.42E-05	1.274503	0.2029
FORECAST	0.810166	0.069423	11.67001	0.0000

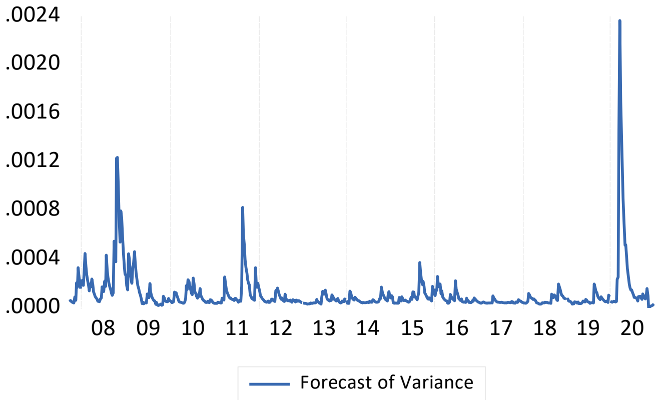
R-squared	0.165844	Mean dependent var	0.000120
Adjusted R-squared	0.164626	S.D. dependent var	0.000322
S.E. of regression	0.000294	Akaike info criterion	-13.42099
Sum squared resid	5.93E-05	Schwarz criterion	-13.40779
Log likelihood	4612.109	Hannan-Quinn criter.	-13.41588
F-statistic	136.1892	Durbin-Watson stat	1.479443
Prob(F-statistic)	0.000000		

Appendix D2. Forecasting using the GJR-GARCH Model

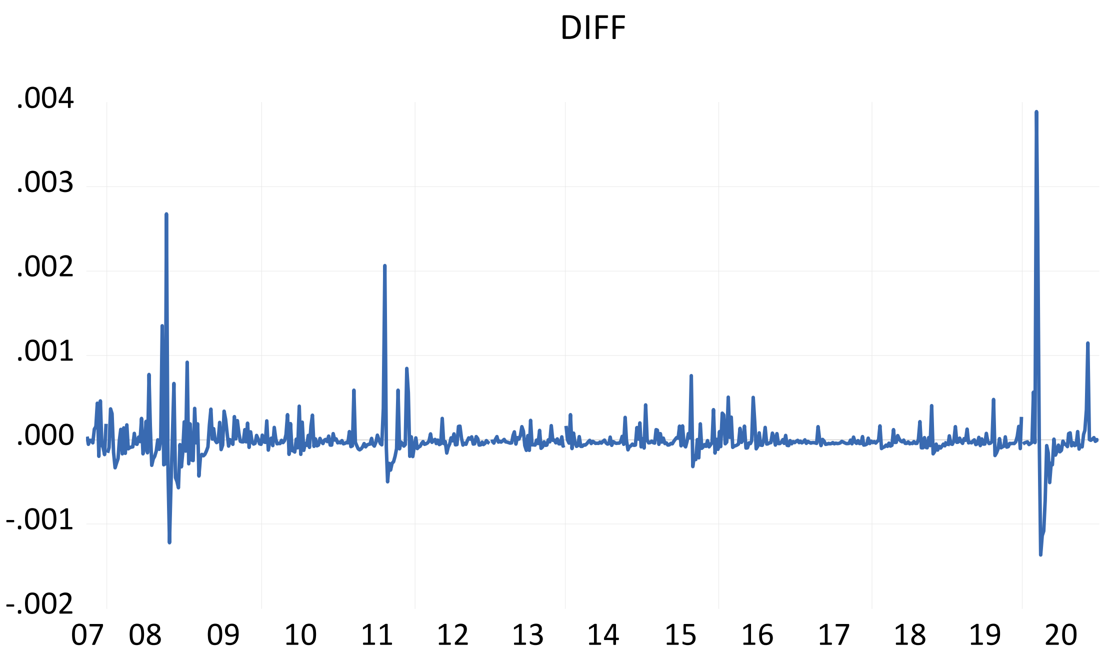
The One-Week-Ahead Forecast of the GJR-GARCH



Forecast: FTSE_100_LF	
Actual: FTSE_100_LOG_RETURNS	
Forecast sample: 9/19/2007 12/30/2020	
Included observations: 687	
Root Mean Squared Error	0.010950
Mean Absolute Error	0.007685
Mean Abs. Percent Error	106.0885
Theil Inequality Coef.	0.987586
Bias Proportion	0.000223
Variance Proportion	NA
Covariance Proportion	NA
Theil U2 Coefficient	0.995922
Symmetric MAPE	190.9836



The Difference between the Realised and Predicted Volatility using E-GARCH



The Regression of Realised against the Predicted Volatility using E-GARCH

Dependent Variable: VOLATILITY
Method: Least Squares
Date: 08/11/21 Time: 17:43
Sample: 9/19/2007 12/30/2020
Included observations: 687

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.52E-05	1.32E-05	1.916366	0.0557
FORECAST	0.778143	0.059393	13.10156	0.0000

R-squared	0.200374	Mean dependent var	0.000120
Adjusted R-squared	0.199207	S.D. dependent var	0.000322
S.E. of regression	0.000288	Akaike info criterion	-13.46327
Sum squared resid	5.69E-05	Schwarz criterion	-13.45007
Log likelihood	4626.632	Hannan-Quinn criter.	-13.45816
F-statistic	171.6509	Durbin-Watson stat	1.470928
Prob(F-statistic)	0.000000		

Appendix E. Normal Market Sub-Period, 2013-2018

Estimation using the GARCH Model

Dependent Variable: FTSE
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 08/11/21 Time: 19:22
Sample: 1/09/2013 12/19/2018
Included observations: 309
Convergence achieved after 11 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000235	0.000408	0.576924	0.5640
Variance Equation				
C	3.18E-06	1.53E-06	2.074756	0.0380
RESID(-1)^2	0.117265	0.040817	2.872968	0.0041
GARCH(-1)	0.837418	0.050969	16.42984	0.0000

R-squared	-0.000083	Mean dependent var	0.000162
Adjusted R-squared	-0.000083	S.D. dependent var	0.008012
S.E. of regression	0.008012	Akaike info criterion	-6.901247
Sum squared resid	0.019771	Schwarz criterion	-6.852919
Log likelihood	1070.243	Hannan-Quinn criter.	-6.881926
Durbin-Watson stat	2.138368		

Estimation using the GJR-GARCH Model

Dependent Variable: FTSE
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 08/11/21 Time: 19:19
Sample: 1/09/2013 12/19/2018
Included observations: 309
Convergence achieved after 18 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-1)^2*(RESID(-1)<0)
+ C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000168	0.000451	-0.371724	0.7101
Variance Equation				
C	4.49E-06	1.68E-06	2.676982	0.0074
RESID(-1)^2	-0.028278	0.034729	-0.814242	0.4155
RESID(-1)^2*(RESID(-1)<0)	0.269042	0.086099	3.124791	0.0018
GARCH(-1)	0.829552	0.050362	16.47191	0.0000

R-squared	-0.001703	Mean dependent var	0.000162
Adjusted R-squared	-0.001703	S.D. dependent var	0.008012
S.E. of regression	0.008019	Akaike info criterion	-6.935556
Sum squared resid	0.019803	Schwarz criterion	-6.875146
Log likelihood	1076.543	Hannan-Quinn criter.	-6.911403
Durbin-Watson stat	2.134909		

Estimation using the E-GARCH Model

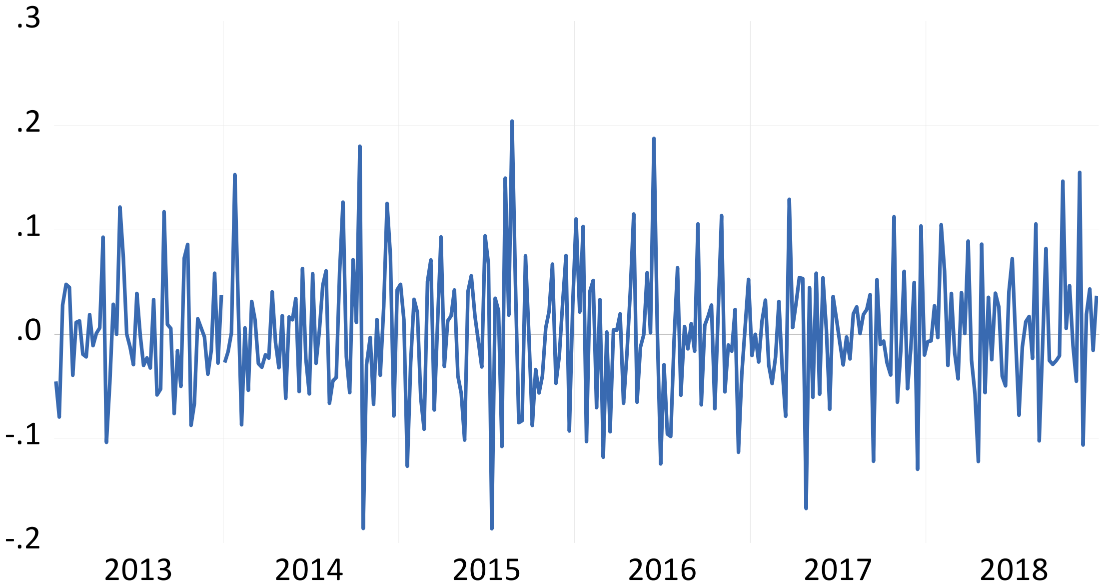
Dependent Variable: FTSE
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 08/11/21 Time: 19:33
Sample: 1/09/2013 12/19/2018
Included observations: 309
Convergence achieved after 20 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(2) + C(3)*ABS(RESID(-1)/@SQRT(GARCH(-1))) +
C(4)*RESID(-1)/@SQRT(GARCH(-1)) + C(5)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000257	0.000447	-0.575230	0.5651
		Variance Equation		
C(2)	-0.852115	0.292082	-2.917385	0.0035
C(3)	0.149732	0.054135	2.765882	0.0057
C(4)	-0.192867	0.050917	-3.787883	0.0002
C(5)	0.924281	0.028654	32.25667	0.0000

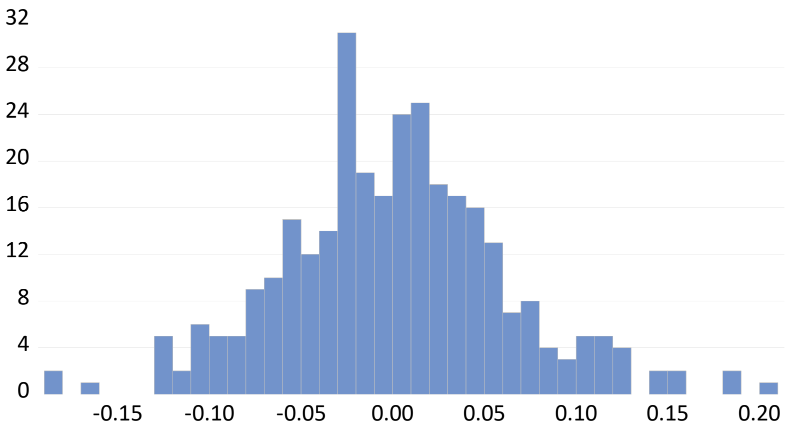
R-squared	-0.002755	Mean dependent var	0.000162
Adjusted R-squared	-0.002755	S.D. dependent var	0.008012
S.E. of regression	0.008023	Akaike info criterion	-6.945835
Sum squared resid	0.019824	Schwarz criterion	-6.885425
Log likelihood	1078.132	Hannan-Quinn criter.	-6.921683
Durbin-Watson stat	2.132669		

Appendix F. Inclusion of the VIX to the Models

Weekly Values of the VIX on the FTSE 100 over 2013-2018



Descriptive Statistics for the VIX Variable



Series: VIX
Sample 1/09/2013 12/19/2018
Observations 309

Mean	0.000487
Median	0.000311
Maximum	0.204347
Minimum	-0.186934
Std. Dev.	0.061826
Skewness	0.167671
Kurtosis	3.673520
Jarque-Bera	7.288339
Probability	0.026143

Stationarity Test for the VIX Variable

Null Hypothesis: VIX has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=15)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-15.81873	0.0000
Test critical values:	1% level	-3.451491	
	5% level	-2.870743	
	10% level	-2.571744	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(VIX)
Method: Least Squares
Date: 08/29/21 Time: 11:00
Sample (adjusted): 1/23/2013 12/19/2018
Included observations: 307 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
VIX(-1)	-1.363917	0.086222	-15.81873	0.0000
D(VIX(-1))	0.163433	0.056366	2.899507	0.0040
C	0.001072	0.003438	0.311825	0.7554

R-squared	0.598623	Mean dependent var	0.000379
Adjusted R-squared	0.595982	S.D. dependent var	0.094770
S.E. of regression	0.060238	Akaike info criterion	-2.771291
Sum squared resid	1.103113	Schwarz criterion	-2.734872
Log likelihood	428.3931	Hannan-Quinn criter.	-2.756727
F-statistic	226.6961	Durbin-Watson stat	2.028366
Prob(F-statistic)	0.000000		

Inclusion of the VIX to the GARCH Model

Dependent Variable: FTSE
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 08/29/21 Time: 23:22
Sample: 1/09/2013 12/19/2018
Included observations: 309
Convergence achieved after 16 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1) + C(5)*VIX

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000833	0.000384	2.170889	0.0299
		Variance Equation		
C	5.60E-06	2.27E-06	2.467898	0.0136
RESID(-1)^2	0.109264	0.048852	2.236631	0.0253
GARCH(-1)	0.775630	0.074853	10.36206	0.0000
VIX	0.000225	3.33E-05	6.751110	0.0000

R-squared	-0.007025	Mean dependent var	0.000162
Adjusted R-squared	-0.007025	S.D. dependent var	0.008012
S.E. of regression	0.008040	Akaike info criterion	-7.024695
Sum squared resid	0.019909	Schwarz criterion	-6.964285
Log likelihood	1090.315	Hannan-Quinn criter.	-7.000543
Durbin-Watson stat	2.123626		

Inclusion of the VIX to the GJR-GARCH Model

Dependent Variable: FTSE
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 08/31/21 Time: 10:26
Sample: 1/09/2013 12/19/2018
Included observations: 309
Convergence achieved after 8 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-1)^2*(RESID(-1)<0)
+ C(5)*GARCH(-1) + C(6)*VIX

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001102	0.000432	2.548702	0.0108
		Variance Equation		
C	1.04E-05	3.49E-06	2.982452	0.0029
RESID(-1)^2	0.084232	0.074619	1.128830	0.2590
RESID(-1)^2*(RESID(-1)<0)	0.128899	0.109288	1.179448	0.2382
GARCH(-1)	0.635735	0.092335	6.885123	0.0000
VIX	0.000246	1.63E-05	15.06289	0.0000

R-squared	-0.013804	Mean dependent var	0.000162
Adjusted R-squared	-0.013804	S.D. dependent var	0.008012
S.E. of regression	0.008067	Akaike info criterion	-7.027737
Sum squared resid	0.020043	Schwarz criterion	-6.955245
Log likelihood	1091.785	Hannan-Quinn criter.	-6.998754
Durbin-Watson stat	2.109426		

Inclusion of the VIX to the E-GARCH Model

Dependent Variable: FTSE
Method: ML ARCH - Normal distribution (Marquardt / EViews legacy)
Date: 08/31/21 Time: 09:48
Sample: 1/09/2013 12/19/2018
Included observations: 309
Convergence achieved after 68 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(2) + C(3)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(4)*RESID(-1)/@SQRT(GARCH(-1)) + C(5)*LOG(GARCH(-1)) + C(6)*VIX

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001161	0.000408	2.844414	0.0044
Variance Equation				
C(2)	-0.259178	0.104806	-2.472939	0.0134
C(3)	0.085892	0.051158	1.678958	0.0932
C(4)	-0.040330	0.053008	-0.760831	0.4468
C(5)	0.980901	0.009888	99.20195	0.0000
C(6)	7.541693	1.198108	6.294670	0.0000

R-squared	-0.015590	Mean dependent var	0.000162
Adjusted R-squared	-0.015590	S.D. dependent var	0.008012
S.E. of regression	0.008074	Akaike info criterion	-7.083543
Sum squared resid	0.020078	Schwarz criterion	-7.011051
Log likelihood	1100.407	Hannan-Quinn criter.	-7.054560
Durbin-Watson stat	2.105716		

Appendix G. Estimation under t-student Distribution Function

Estimation of GARCH under t-student Distribution Function

Dependent Variable: FTSE
Method: ML ARCH - Student's t distribution (Marquardt / EViews legacy)
Date: 08/31/21 Time: 09:24
Sample: 1/09/2013 12/19/2018
Included observations: 309
Convergence achieved after 44 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000337	0.000390	0.863627	0.3878
		Variance Equation		
C	3.33E-06	2.12E-06	1.566661	0.1172
RESID(-1)^2	0.127934	0.057693	2.217501	0.0266
GARCH(-1)	0.827486	0.068972	11.99737	0.0000
T-DIST. DOF	8.207039	5.076723	1.616602	0.1060

R-squared	-0.000474	Mean dependent var	0.000162
Adjusted R-squared	-0.000474	S.D. dependent var	0.008012
S.E. of regression	0.008014	Akaike info criterion	-6.912111
Sum squared resid	0.019779	Schwarz criterion	-6.851701
Log likelihood	1072.921	Hannan-Quinn criter.	-6.887959
Durbin-Watson stat	2.137531		

Estimation of GJR-GARCH under t-student Function

Dependent Variable: FTSE
Method: ML ARCH - Student's t distribution (Marquardt / EViews legacy)
Date: 08/31/21 Time: 09:35
Sample: 1/09/2013 12/19/2018
Included observations: 309
Convergence achieved after 57 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-1)^2*(RESID(-1)<0) + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	7.97E-05	0.000413	0.193083	0.8469
		Variance Equation		
C	1.06E-05	5.07E-06	2.085753	0.0370
RESID(-1)^2	-0.062899	0.058428	-1.076509	0.2817
RESID(-1)^2*(RESID(-1)<0)	0.443059	0.180099	2.460088	0.0139
GARCH(-1)	0.684772	0.118828	5.762730	0.0000
T-DIST. DOF	8.349478	4.922459	1.696201	0.0898

R-squared	-0.000107	Mean dependent var	0.000162
Adjusted R-squared	-0.000107	S.D. dependent var	0.008012
S.E. of regression	0.008012	Akaike info criterion	-6.942766
Sum squared resid	0.019772	Schwarz criterion	-6.870274
Log likelihood	1078.657	Hannan-Quinn criter.	-6.913784
Durbin-Watson stat	2.138316		

Estimation of E-GARCH under t-student Distribution Function

Dependent Variable: FTSE
Method: ML ARCH - Student's t distribution (Marquardt / EViews legacy)
Date: 08/29/21 Time: 18:26
Sample: 1/09/2013 12/19/2018
Included observations: 309
Convergence achieved after 147 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(2) + C(3)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(4)*RESID(-1)/@SQRT(GARCH(-1)) + C(5)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-6.39E-05	0.000419	-0.152367	0.8789
Variance Equation				
C(2)	-1.024361	0.417090	-2.455971	0.0141
C(3)	0.172547	0.080322	2.148189	0.0317
C(4)	-0.220715	0.069036	-3.197098	0.0014
C(5)	0.909076	0.040617	22.38179	0.0000
T-DIST. DOF	10.80971	8.183106	1.320979	0.1865

R-squared	-0.000801	Mean dependent var	0.000162
Adjusted R-squared	-0.000801	S.D. dependent var	0.008012
S.E. of regression	0.008015	Akaike info criterion	-6.949476
Sum squared resid	0.019786	Schwarz criterion	-6.876984
Log likelihood	1079.694	Hannan-Quinn criter.	-6.920494
Durbin-Watson stat	2.136834		

Appendix H. Estimation Jointly with VIX and t-student

GARCH Estimation Jointly with the VIX and t-student

Dependent Variable: FTSE
Method: ML ARCH - Student's t distribution (Marquardt / EViews legacy)
Date: 08/29/21 Time: 23:19
Sample: 1/09/2013 12/19/2018
Included observations: 309
Convergence achieved after 42 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1) + C(5)*VIX

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001044	0.000402	2.594307	0.0095
		Variance Equation		
C	8.21E-06	1.37E-06	5.985448	0.0000
RESID(-1)^2	0.150575	0.048408	3.110541	0.0019
GARCH(-1)	0.696883	0.038922	17.90471	0.0000
VIX	0.000248	4.75E-05	5.226450	0.0000
T-DIST. DOF	20.00002	26.36500	0.758582	0.4481

R-squared	-0.012138	Mean dependent var	0.000162
Adjusted R-squared	-0.012138	S.D. dependent var	0.008012
S.E. of regression	0.008060	Akaike info criterion	-7.006323
Sum squared resid	0.020010	Schwarz criterion	-6.933831
Log likelihood	1088.477	Hannan-Quinn criter.	-6.977341
Durbin-Watson stat	2.112899		

GJR-GARCH Estimation Jointly with the VIX and t-student

Dependent Variable: FTSE
Method: ML ARCH - Student's t distribution (Marquardt / EViews legacy)
Date: 08/29/21 Time: 23:25
Sample: 1/09/2013 12/19/2018
Included observations: 309
Convergence achieved after 8 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-1)^2*(RESID(-1)<0) + C(5)*GARCH(-1) + C(6)*VIX

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000992	0.000392	2.530591	0.0114
		Variance Equation		
C	9.78E-06	3.93E-06	2.489162	0.0128
RESID(-1)^2	0.067025	0.074626	0.898145	0.3691
RESID(-1)^2*(RESID(-1)<0)	0.159936	0.116718	1.370281	0.1706
GARCH(-1)	0.637625	0.113687	5.608598	0.0000
VIX	0.000237	1.03E-05	23.04768	0.0000
T-DIST. DOF	20.00002	24.27620	0.823853	0.4100

R-squared	-0.010748	Mean dependent var	0.000162
Adjusted R-squared	-0.010748	S.D. dependent var	0.008012
S.E. of regression	0.008055	Akaike info criterion	-7.025845
Sum squared resid	0.019982	Schwarz criterion	-6.941271
Log likelihood	1092.493	Hannan-Quinn criter.	-6.992032
Durbin-Watson stat	2.115803		

E-GARCH Estimation Jointly with VIX and t-student

Dependent Variable: FTSE
Method: ML ARCH - Student's t distribution (Marquardt / EViews legacy)
Date: 08/31/21 Time: 09:28
Sample: 1/09/2013 12/19/2018
Included observations: 309
Convergence achieved after 49 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(2) + C(3)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(4)*RESID(-1)/@SQRT(GARCH(-1)) + C(5)*LOG(GARCH(-1)) + C(6)*VIX

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001159	0.000408	2.843648	0.0045
		Variance Equation		
C(2)	-0.260756	0.110396	-2.361999	0.0182
C(3)	0.086211	0.052573	1.639852	0.1010
C(4)	-0.040229	0.053696	-0.749199	0.4537
C(5)	0.980764	0.010321	95.02591	0.0000
C(6)	7.551732	1.241039	6.085007	0.0000
T-DIST. DOF	340.7380	9524.731	0.035774	0.9715

R-squared	-0.015534	Mean dependent var	0.000162
Adjusted R-squared	-0.015534	S.D. dependent var	0.008012
S.E. of regression	0.008074	Akaike info criterion	-7.076770
Sum squared resid	0.020077	Schwarz criterion	-6.992196
Log likelihood	1100.361	Hannan-Quinn criter.	-7.042957
Durbin-Watson stat	2.105833		